HAPPY LEARNING
Starts Today

15UEC904

LINEAR CONTROL ENGINEERING

III YR/V SEM

DEPARTMENT OF ECE

Presented by

Dr.M.Parisa Beham

Asso.Prof./ECE

INSTITUTE VISION & MISSION

Institute Vision	**To promote excellence in technical education and scientific research for the benefit of the society**
Institute Mission	• To provide quality technical education to fulfill the aspiration of the student and to meet the needs of the Industry • To provide holistic learning ambience • To impart skills leading to employability and entrepreneurship • To establish effective linkage with industries • To promote Research and Development activities • To offer services for the development of society through education and technology

DEPARTMENT VISION & MISSION

Department Vision (ECE)	**To achieve excellence in education and research in the field of Electronics and Communication Engineering for the development of society.**
Department Mission (ECE)	• Imparting quality technical education in Electronics and Communication Engineering through contemporary laboratory facilities and accomplished faculty to cater to the needs of the industry. • Providing a conducive learning environment through the state of the art infrastructure and innovative teaching learning practices. • Infusing the professional skills needed for employability and entrepreneurship. • Collaborating with industries for mutual benefit of knowledge transfer. • Promoting research in Electronics and Communication Engineering. • Providing services to the society through extension activities and technology enabled services.

PROGRAM EDUCATIONAL OBJECTIVES

PROGRAMME EDUCATIONAL OBJECTIVES	
PEO – I	Possess strong technical knowledge in Electronics and Communication Engineering to address the real world challenges (Core Competence)
PEO – II	Demonstrate continual interest to learn new technologies for successful professional career (Lifelong Learning)
PEO – III	Exhibit professional skills and practice ethical principles with social consciousness (Professionalism)

PROGRAM SPECIFIC OBJECTIVES

PROGRAMME SPECIFIC OUTCOMES	
PSO – I	Design and Develop solution in the field of Signal processing and Communication
PSO – II	Demonstrate competency in the design and development of Embedded / VLSI systems

PROGRAM OUTCOMES

(1)	**Apply the knowledge of mathematics, science, engineering fundamentals, and Electronics and Communication engineering to solve complex engineering problems. (Engineering knowledge)**
(2)	Identify, formulate, review research literature, and analyze complex Electronics and Communication engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences. (***Problem Analysis***)
(3)	Design solutions for complex Electronics and Communication engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations (**Design and Development of Solutions**)
(4)	Conduct investigations of complex Electronics and Communication Engineering problems using research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions. **(Investigation of Complex Problems**).
(5)	Select, and apply appropriate techniques, resources, and modern engineering and IT tools for prediction, modeling and simulation of complex Electronics and Communication Engineering activities with an understanding of the limitations. (**Modern Engineering Tools**).
(6)	Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice. (**Engineer and Society**).

PROGRAM OUTCOMES – Contd..

(7)	**Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development. (Environment and Sustainability)**
(8)	Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice. (**Ethics)**
(9)	Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings. **(Individual and Team Work**).
(10)	Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions. (**Communication**).
(11)	Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.(**Project Management and Finance**)
(12)	Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change. (**Life-long learning**)..

COURSE DETAILS

Course Code/Course Name		15UEC404 – Signals and Systems
Course Coordinator		Dr. M. Parisa Beham
Course Instructors	III yr A-Sec	Dr.K A Shahul Hameed , Prof./ECE
	III yr B-Sec	Mr. B.Muthupandian, Asst.Prof./ECE
	III yr C-Sec	Dr. M.Parisa Beham, Asso.Prof./ECE

OBJECTIVES

- To introduce the concept of open loop and closed loop (feedback) systems
- To provide knowledge of time domain and frequency domain analysis of control systems required for stability analysis
- To present the compensation technique that can be used to stabilize control systems

15UEC904 – LINEAR CONTROL ENGINEERING

REGULATION - 2015

L	T	P	C
3	0	0	3

45 periods

UNIT I - CONTROL SYSTEM MODELING

Basic Elements of Control System – Open loop and Closed loop systems - Differential equation - Transfer function, Modeling of Electric systems, Translational and rotational mechanical systems - Block diagram reduction Techniques - Signal flow graph (9)

UNIT II - TIME RESPONSE ANALYSIS

Time response analysis - First Order Systems - Impulse and Step Response analysis of second order systems - Steady state errors – P, PI, PD and PID Compensation, Analysis using MATLAB (9)

UNIT III - FREQUENCY RESPONSE ANALYSIS

Frequency Response- Bode Plot, Polar Plot, Nyquist Plot - Frequency Domain specifications from the plots - Series, Parallel, series-parallel Compensators- Lead, Lag, and Lead Lag Compensators, Analysis using MATLAB. (9)

UNIT IV - STABILITY ANALYSIS

Stability, Routh-Hurwitz Criterion, Root Locus Technique, Construction of Root Locus, Stability, Dominant Poles, Application of Root Locus Diagram - Nyquist Stability Criterion- Relative Stability, Analysis using MATLAB. (9)

UNIT V - STATE VARIABLE ANALYSIS

State space representation of Continuous Time systems – State equations – Transfer function from State Variable Representation – Solutions of the state equations - Concepts of Controllability and Observability – State space representation for discrete time systems. **(9)**

BOOKS TO BE REFERRED

TEXT BOOKS

1. **J.Nagrath, M.Gopal ”Control Systems: Engineering ”, Anshan Publishers, 5thEdition, 2008.**

2. **M.Gopal, ”Control Systems: Principles and Design ”, Tata McGraw Hill, 4th Edition, 2012.**

REFERENCE BOOKS:

1. **M.Gopal, ” Digital Control and State Variable Methods ”, TMH, 2nd Edition, 2007.**
2. **Schaum‟s Outline Series, ” Feedback and Control Systems ”, Tata McGraw-Hill, 2007.**
3. **Richard C. Dorf, Robert H. Bishop, “Modern Control Systems”, Addidon – Wesley, 9th Edition,2010.**
4. **Benjamin.C.Kuo , “Automatic control systems”, Prentice Hall of India, 6thEdition ,2013.**
5. **John J.D‟azzo , Constantine H.Houpis , “Linear control system analysis and design”, Tata McGrow-Hill, 1995.**

COURSE OUTCOMES (CO)

CO.1	**Develop mathematical models for Electrical and Mechanical systems. (K3 - Apply)**
CO.2	**Analyze the time response of first and Second order systems. (K4 - ANALYZE)**
CO.3	**Analyze the LTI systems through various frequency response plots. (K4 - ANALYZE)**
CO.4	**Analyze stability of systems using analytical and graphical methods. (K4 - ANALYZE)**
CO.5	**Analyze the MIMO systems using state space model (K4- ANALYZE)**

COURSE ARTICULATION MATRIX

CO	POs												PSOs	
	1	2	3	4	5	6	7	8	9	10	11	12	I	II
CO.1	3	2	2	-	-	-	-	-	-	-	-	2	-	2
CO.2	3	3	2	-	2	-	-	-	-	-	-	2	2	-
CO.3	3	3	2	-	2	-	-	-	-	-	-	2	-	2
CO.4	3	3	2	-	2	-	-	-	-	-	-	2	2	2
CO.5	3	3	2	-	-	-	-	-	-	-	-	2	2	2
CAM	3	3	2	-	2	-	-	-	-	-	-	2	2	2

3- Strong   2- Medium   1- Weak

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REGULATION - 2015

L	T	P	C
3	0	0	3

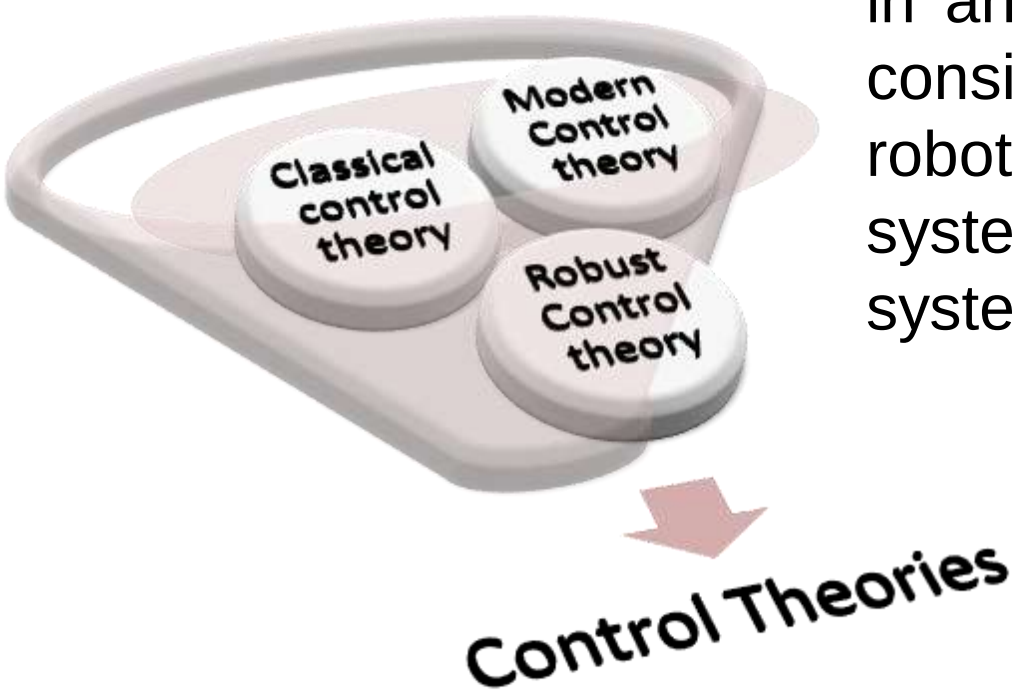
45 periods

CO.1	**Develop mathematical models for Electrical and Mechanical systems. (K3 - Apply)**

UNIT I - CONTROL SYSTEM MODELING

Basic Elements of Control System – Open loop and Closed loop systems - Differential equation - Transfer function, Modeling of Electric systems, Translational and rotational mechanical systems - Block diagram reduction Techniques - Signal flow graph (9)

Introduction to Control System



Automatic control is essential in any field of engineering and considered as integral part of robotic systems, space vehicle systems, modern manufacturing systems etc.

Terminologies

System – An interconnection of elements and devices for a desired purpose.

Control System – An interconnection of components forming a system configuration that will provide a desired response.

Plant - A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.

Process – The device, plant, or system under control. The input and output relationship represents the cause-and-effect relationship of the process.

Process to be controlled.

Open Loop and Closed Loop Systems

OPEN LOOP SYSTEM

Any physical system which does not automatically correct the variation in its output, is called an ***open loop system***, or control system in which the output quantity has no effect upon the input quantity are called open-loop control system. This means that the output is not fedback to the input for correction.

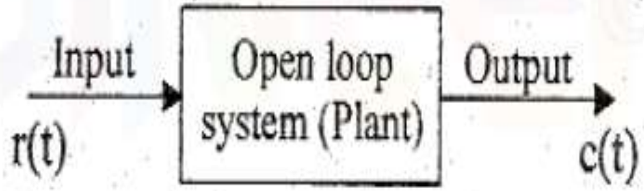


Fig 1.1 : Open loop system.

In open loop system the output can be varied by varying the input. But due to external disturbances the system output may change. When the output changes due to disturbances, it is not followed by changes in input to correct the output. In open loop systems the changes in output are corrected by changing the input manually.

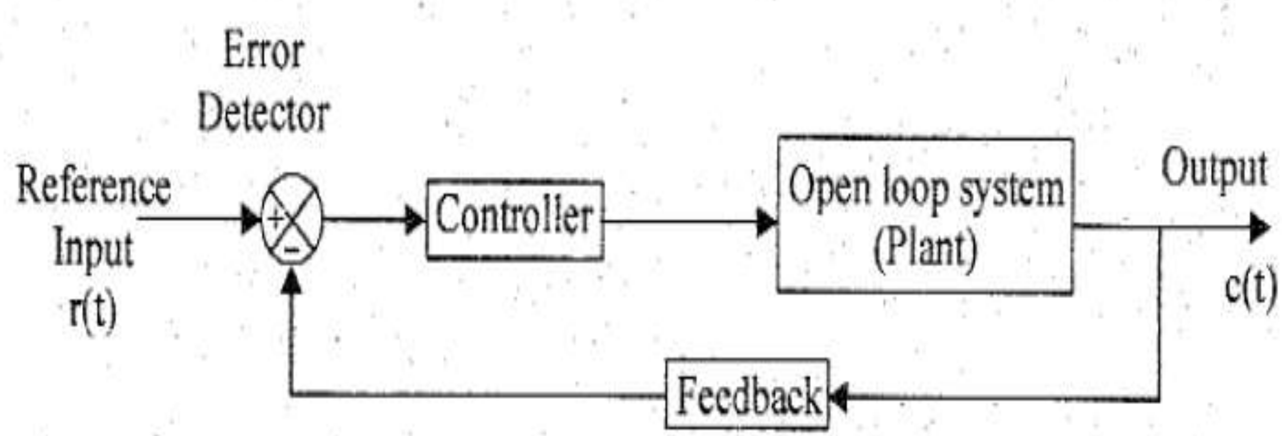
Closed Loop Systems

Closed-Loop Control Systems. Feedback control systems are often referred to as *closed-loop control systems.*

In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal, is fed to the controller so as to reduce the error and bring the output of the system to a desired value.

CLOSED LOOP SYSTEM

Control systems in which the output has an effect upon the input quantity in order to maintain the desired output value are called ***closed loop systems.***



Example:Open Loop Control Systems

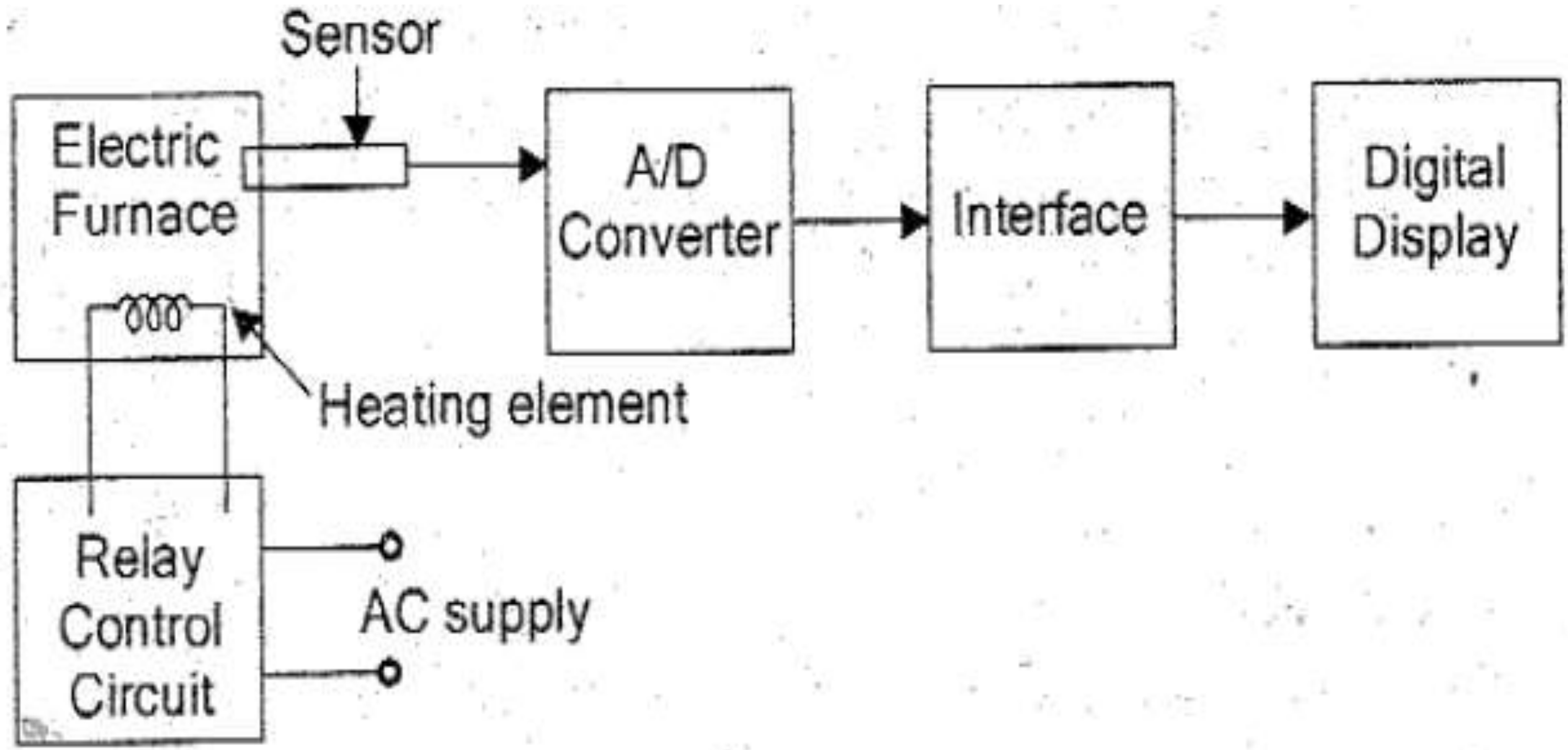


Fig 1.3 : Open loop temperature control system.

Example: Closed Loop Control Systems

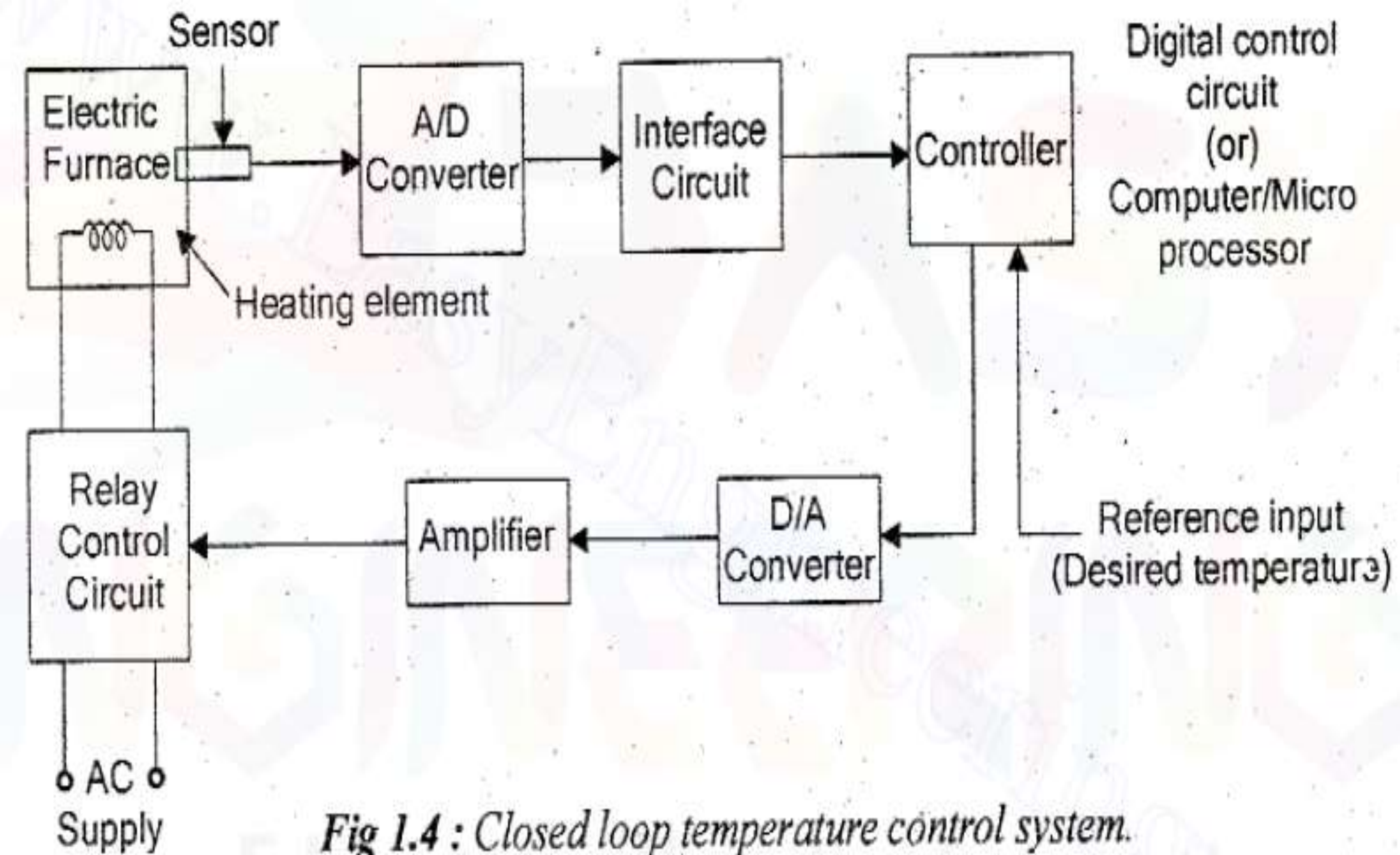


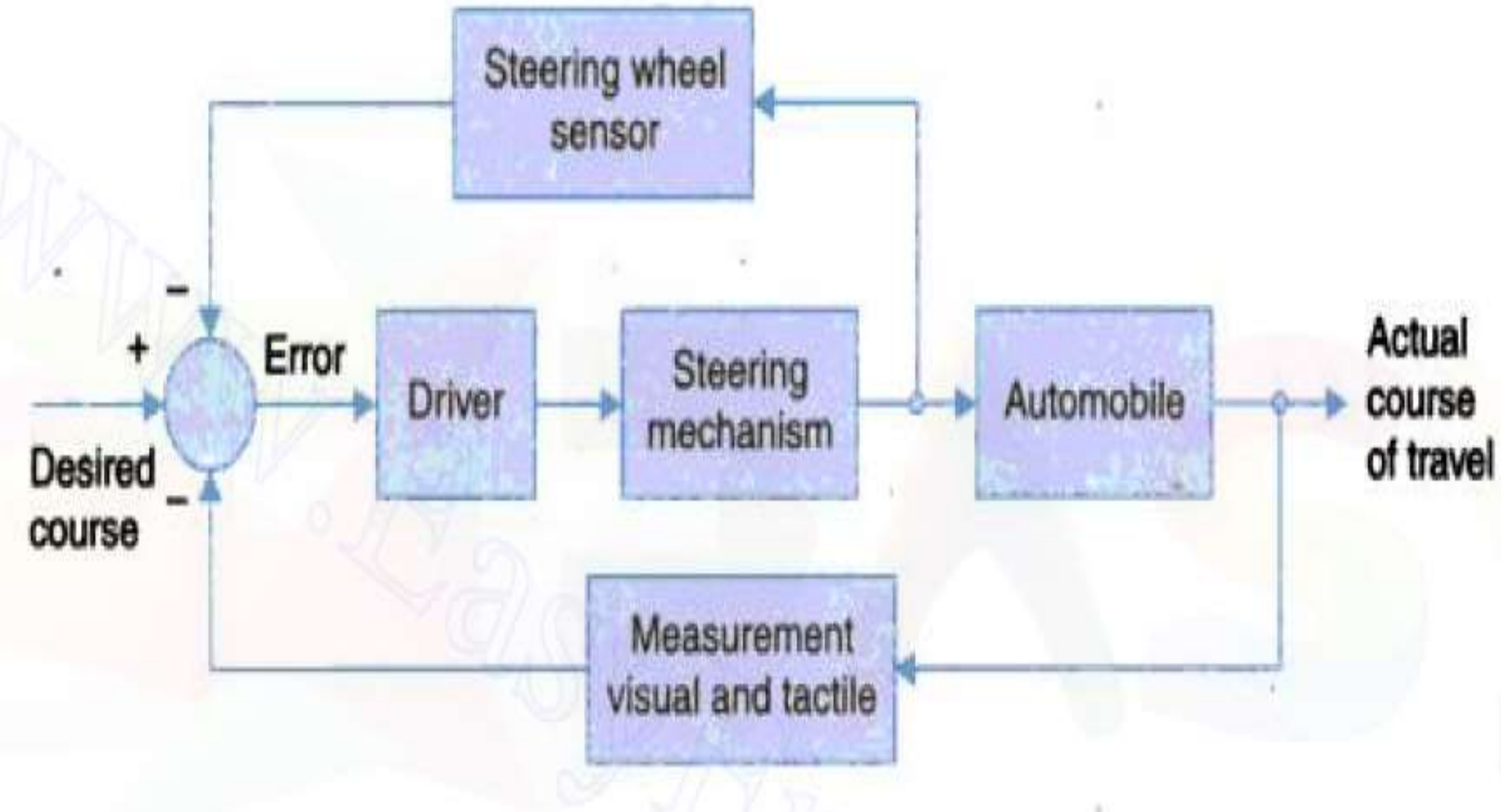
Fig 1.4 : Closed loop temperature control system.

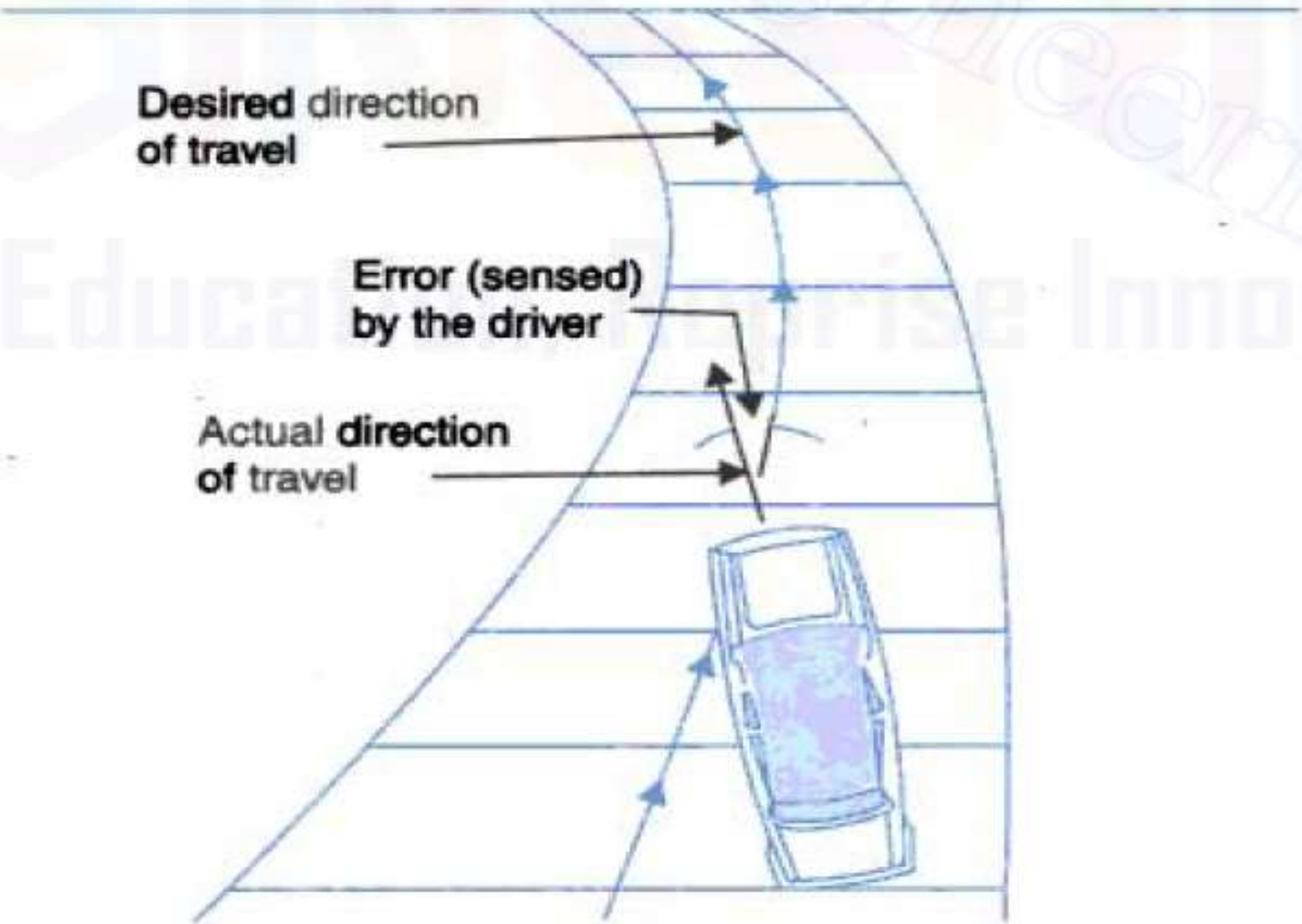
Open Loop and Closed Loop Systems

- Simple construction and ease of maintenance.
- Less expensive than a corresponding closed-loop system.
- There is no stability problem.
- They are inaccurate and unreliable
- The effect of external disturbance signals can be made very small.

- They are more complex and expensive
- Cost of maintenance is high
- The systems are prone to instability. Oscillations in the output many occur.
- They are more accurate and reliable.
- If external disturbances are present, output differs significantly from the desired value.

Real Time Example

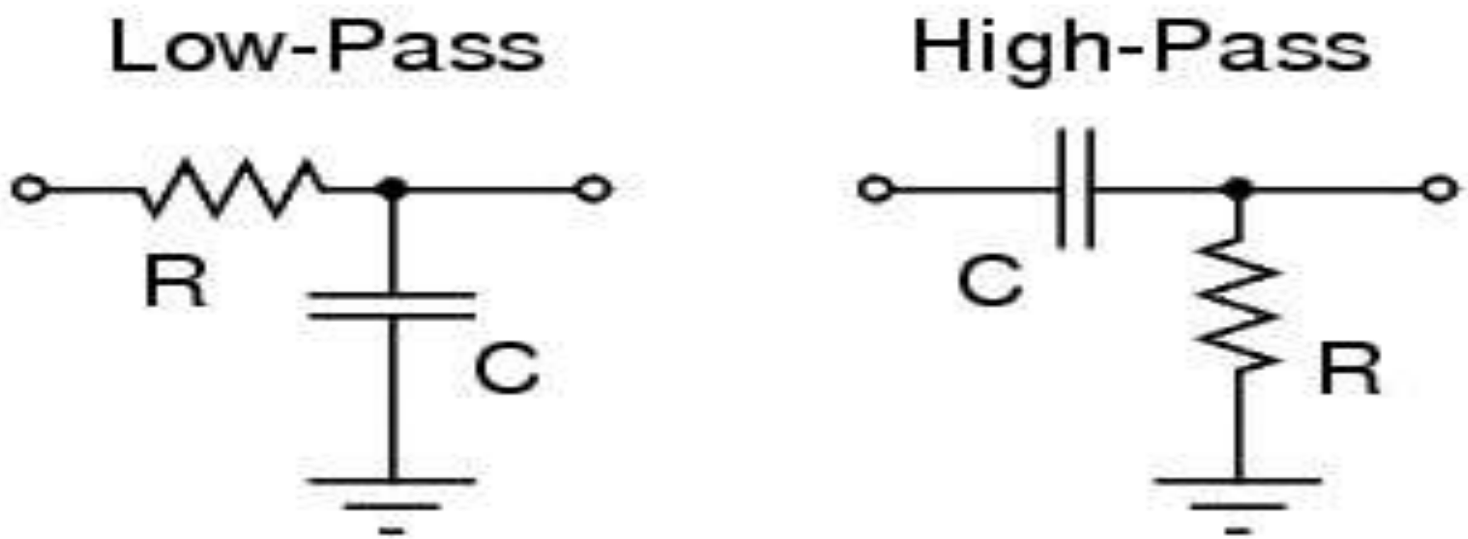



Desired direction of travel
Error (sensed) by the driver
Actual direction of travel

MATHEMATICAL MODELS OF PHYSICAL SYSTEMS

Introduction

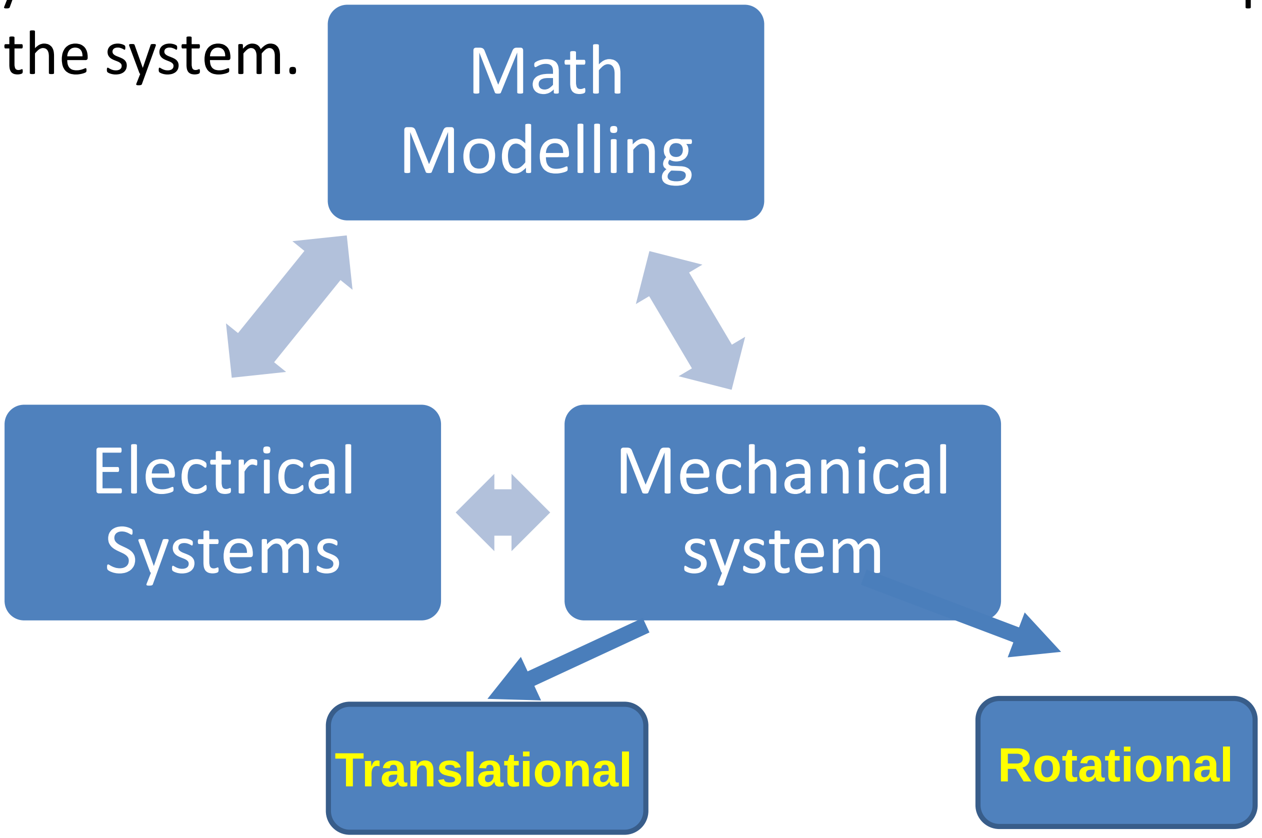
- Idealizing assumptions are always made for the purpose of analysis and synthesis of systems
- An idealized physical systems are called as physical model



$$\text{Transfer function} = \left.\frac{\text{Laplace Transform of output}}{\text{Laplace Transform of input}}\right|_{\text{with zero initial conditions}}$$

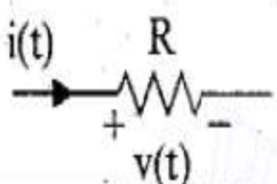
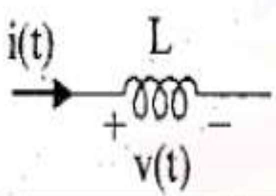
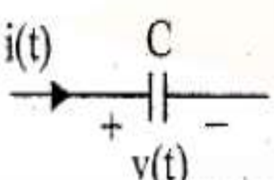
Mathematical model

- After obtaining the physical model, need to generate a mathematical model
- Mathematical models of physical systems are key elements in the design and analysis of control systems
- Use of appropriate physical laws such as Kirchoff's law , Coulombs law etc.
- A control system can be modelled as a scalar differential equation describing the system.



Mathematical Modelling of Electrical Systems

Differential Equations – By Kirchoff's voltage law and current law

Element	Voltage across the element	Current through the element
	$v(t) = Ri(t)$	$i(t) = \frac{v(t)}{R}$
	$v(t) = L\frac{d}{dt}i(t)$	$i(t) = \frac{1}{L}\int v(t)\,dt$
	$v(t) = \frac{1}{C}\int i(t)\,dt$	$i(t) = C\frac{dv(t)}{dt}$

Ex.1 Obtain the transfer function for the electrical network shown

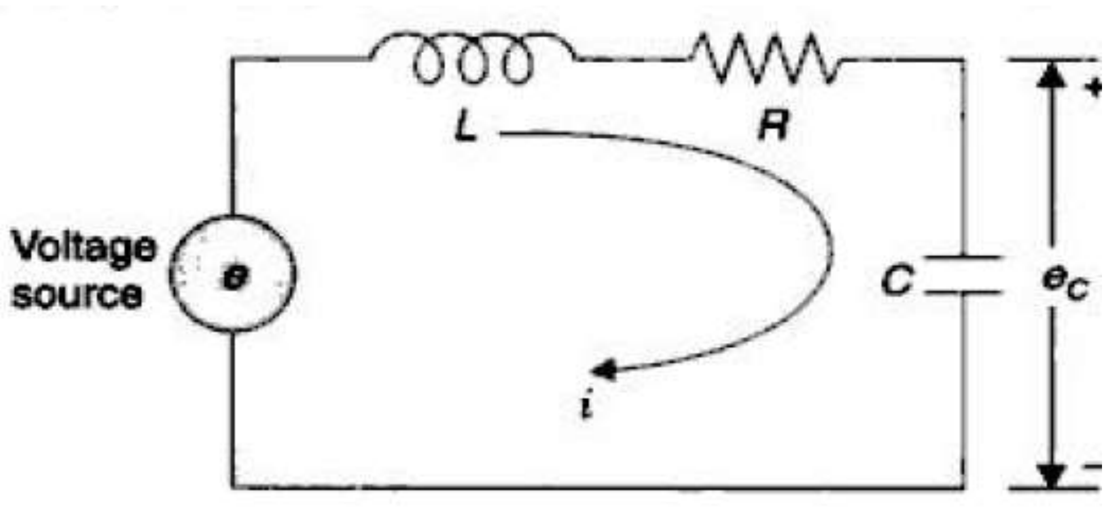
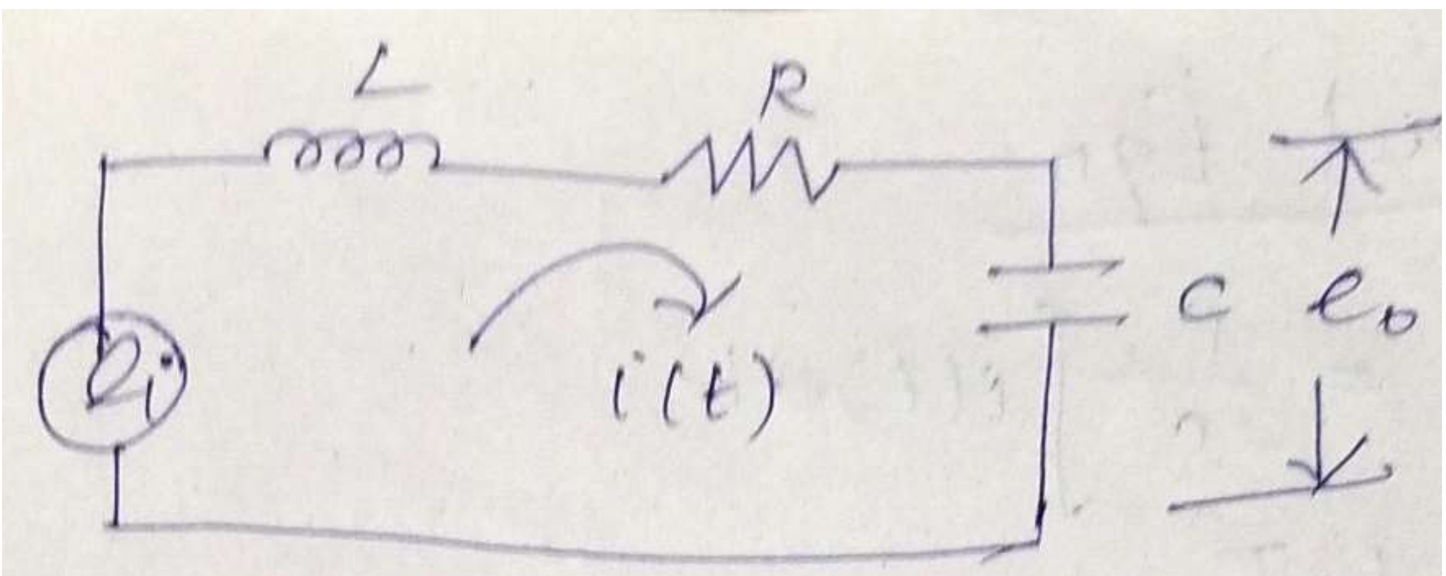
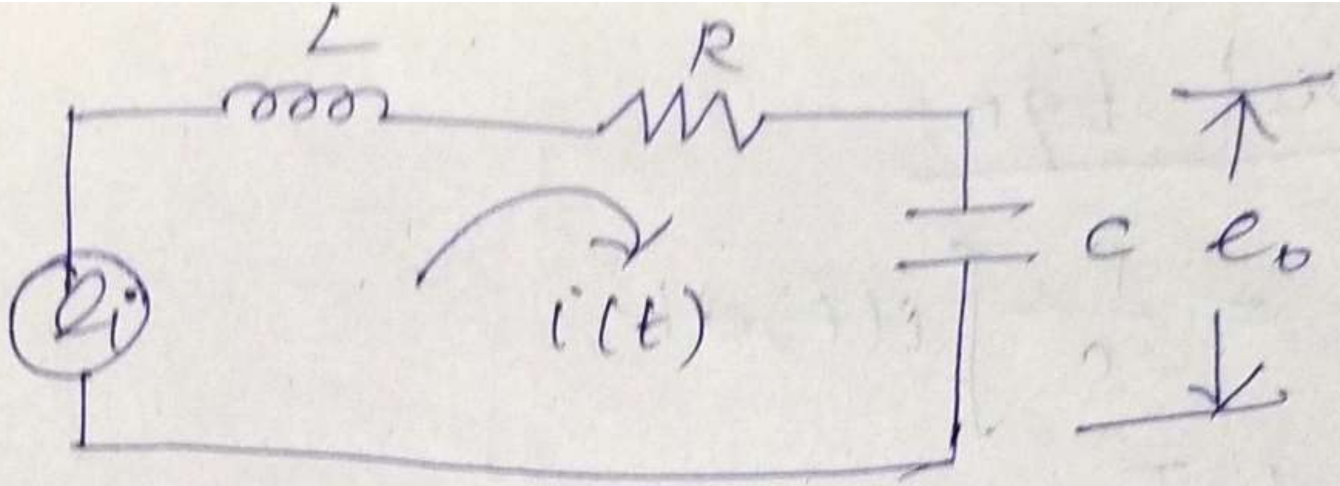


Fig. 2.12. L-R-C series circuit.



Ex:1 Contd..



Apply K.V.L :

Input:

$$e_i = V_L + V_R + V_C$$

$$e_i = L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

Ex:1 Contd..

$$e_i = L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

Apply Laplace Transform: w.k.that

On e_i

$$L\left[\frac{di(t)}{dt}\right] = sI(s)$$

$$L\left[\int i(t)\,dt\right] = \frac{I(s)}{s}$$

$$L[i(t)] = I(s)$$

$$\Rightarrow e_i(s) = LsI(s) + RI(s) + \frac{I(s)}{C\cdot s} \rightarrow ①$$

$$e_i(s) = I(s)\left[Ls + R + \frac{1}{Cs}\right] \rightarrow ①$$

Ex:1 Contd..

Output Eqn:

$$e_0 = \frac{1}{C}\int i(t)\,dt$$

Apply L.T

$$e_0(s) = \frac{1}{C}\frac{I(s)}{s} \rightarrow (2)$$

Transfer function

$$\frac{e_0(s)}{e_i(s)} = \frac{1/Cs \cdot I(s)}{I(s)\left[Ls + R + 1/Cs\right]}$$

Ex:1 Contd..

$$= \frac{1/\cancel{C}s}{\frac{LCs^2 + RCs + 1}{\cancel{C}s}}$$

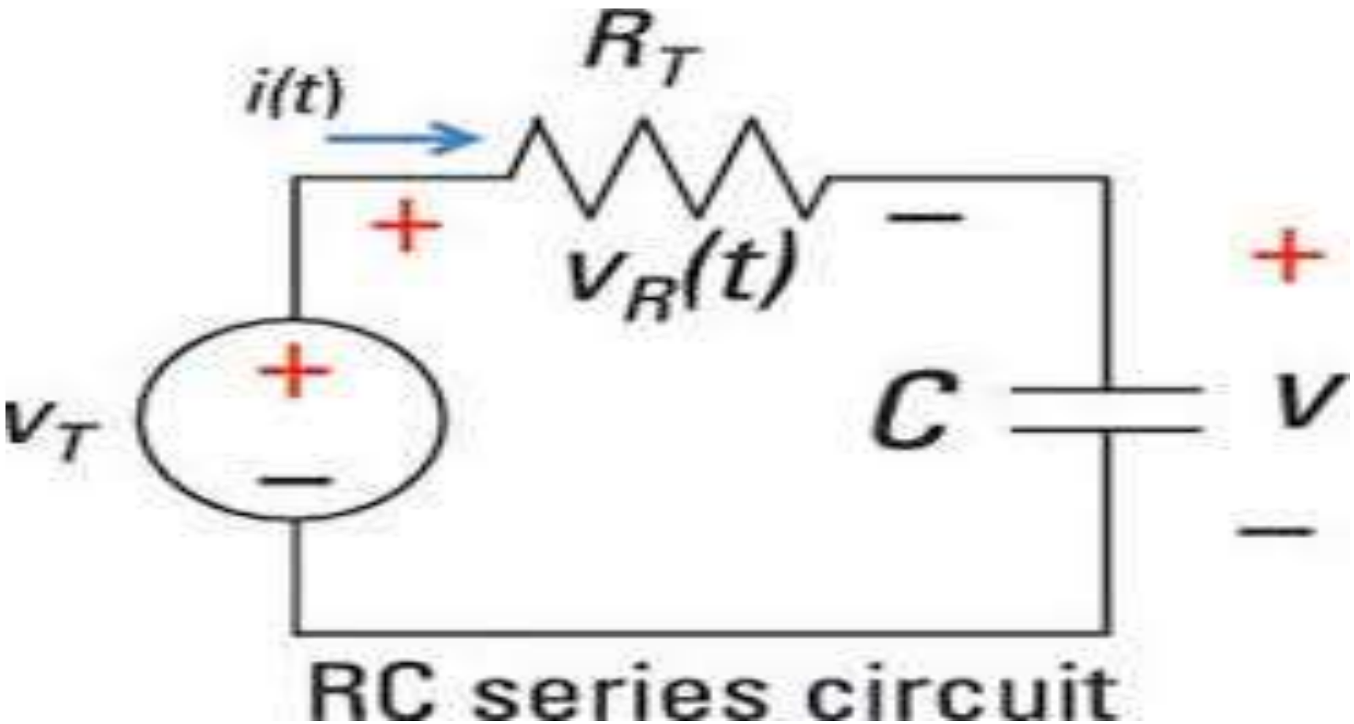
$$= \frac{1}{LCs^2 + RCs + 1}$$

$$= \frac{1}{LC\left[s^2 + \frac{RC}{LC}s + \frac{1}{LC}\right]}$$

$$\Rightarrow \frac{e_o(s)}{e_i(s)} = \frac{1/LC}{s^2 + R/L\, s + 1/LC}$$

Exercise Problem 1:

Obtain the transfer function for the electrical network shown.



Input Diff'tl Eqn:

$$V_T = V_R(t) + V_C(t)$$

$$V_T = R_T \cdot i(t) + \frac{1}{C}\int i(t)\,dt$$

Take L.T.

$$V_T(s) = R_T I(s) + \frac{1}{C}\cdot\frac{I(s)}{s}$$

$$\Rightarrow \boxed{V_T(s) = I(s)\left[R_T + \frac{1}{Cs}\right]} \longrightarrow (1)$$

Output Equation:

$$V = \frac{1}{C}\int i(t)\,dt$$

$\xrightarrow{L.T}$

$$V(s) = \frac{1}{C}\frac{I(s)}{s} \longrightarrow (2)$$

Tr. fn:

$$\frac{V(s)}{V_T(s)} = \frac{\frac{1}{Cs} \cdot I(s)}{I(s)\left[R_T + \frac{1}{Cs}\right]} = \frac{\frac{1}{Cs}}{R_T Cs + 1 / Cs}$$

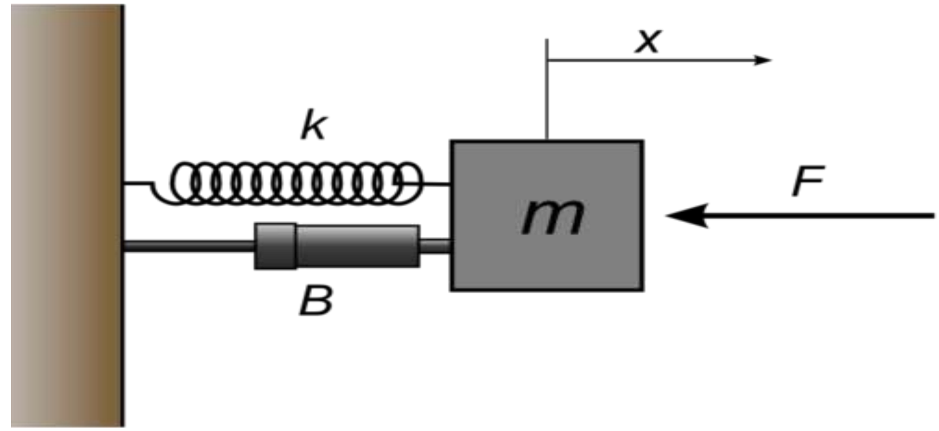
$$= \frac{1}{R_T Cs + 1} = \frac{1}{R_T C\left[s + \frac{1}{R_T C}\right]}$$

$$\frac{V(s)}{V_T(s)} = \frac{1/R_T C}{\left[s + \frac{1}{R_T C}\right]}$$

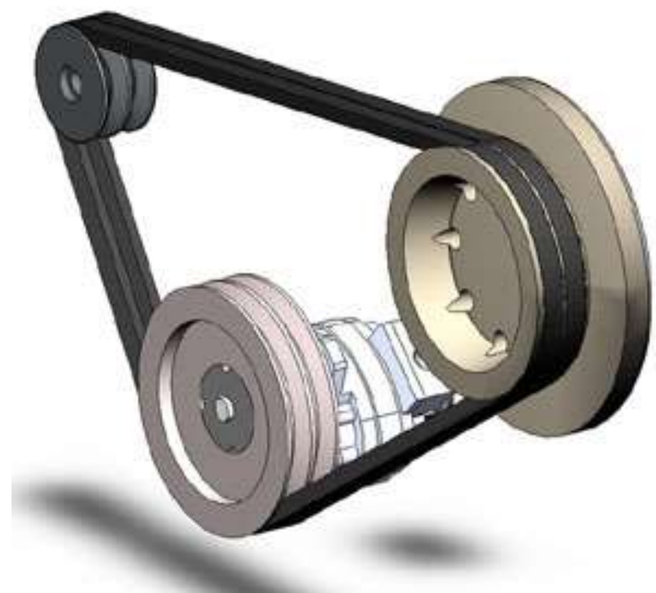
MATHEMATICAL MODEL OF MECHANICAL SYSTEMS

Basic Types of Mechanical Systems

- Translational
  - Linear Motion



- Rotational
  - Rotational Motion

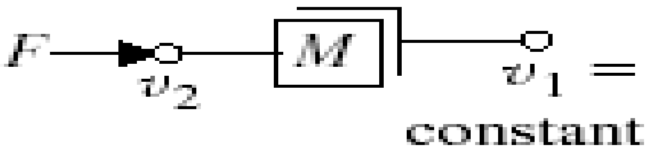


Basic Elements of Translational Mechanical Systems

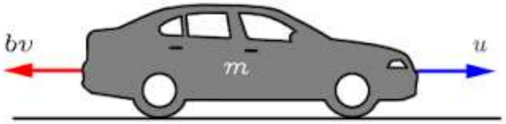
i

Translational Mass

M



Weight of the mechanical system



ii)

Translational Spring

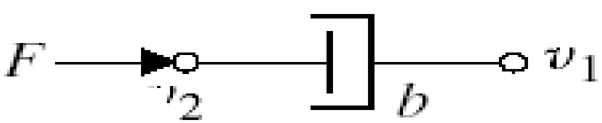
K

Elastic Deformation of the body

Translational Dash-pot

B

iii)



Friction existing in Mech system

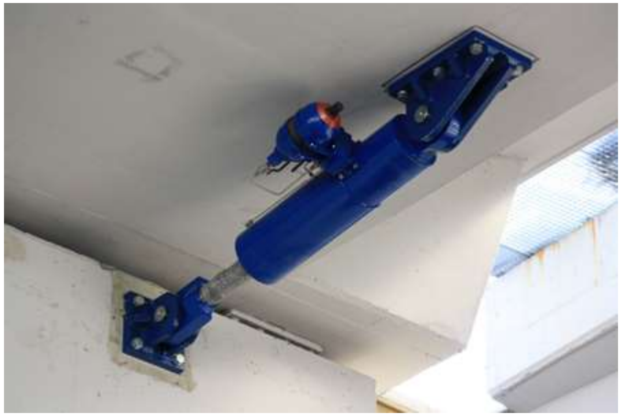
Common Uses of Dashpots

Door Stoppers

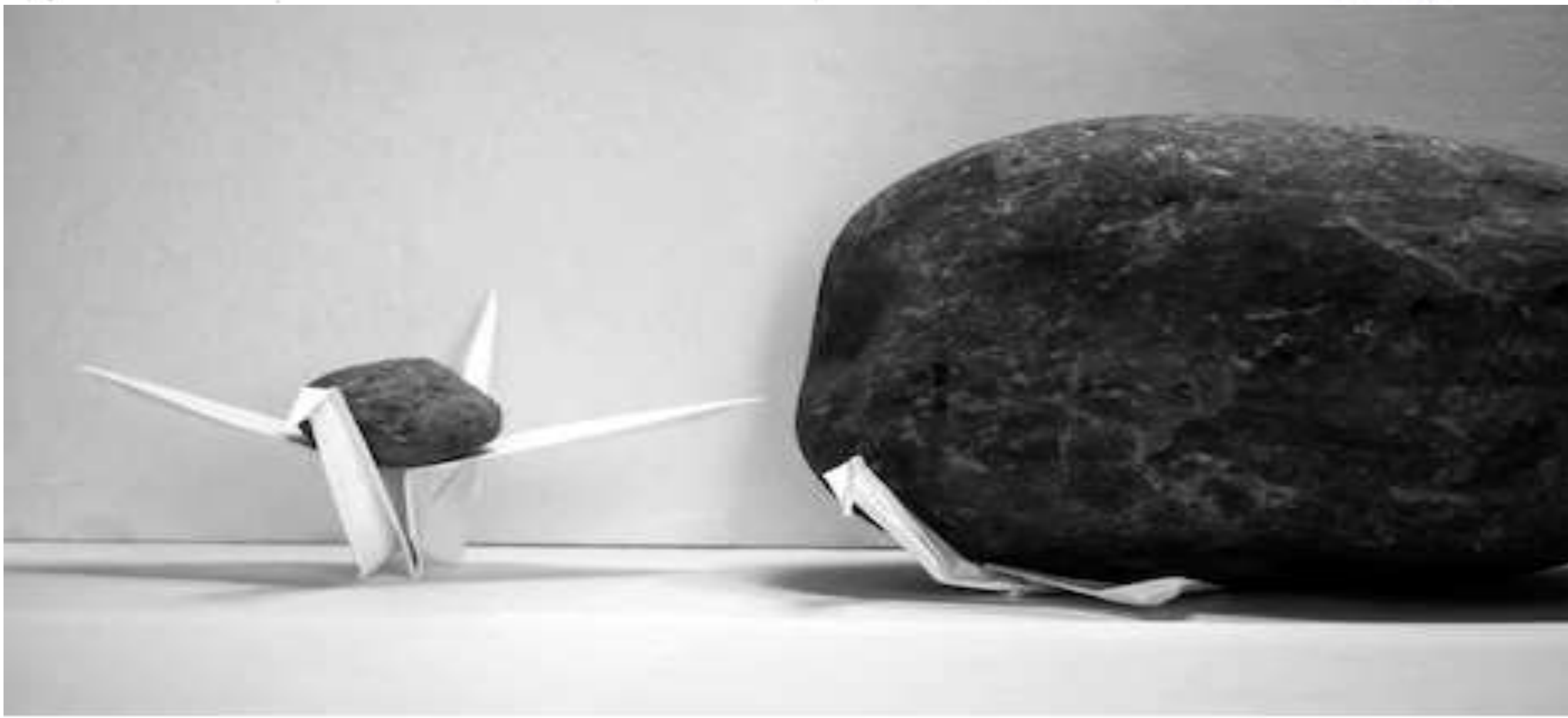
Vehicle Suspension

Bridge Suspension

Flyover Suspension



When a force is applied to a translational mechanical system, it is opposed by opposing forces due to mass, friction and elasticity of the system. The force acting on a mechanical body are governed by ***Newton's second law of motion.*** For translational systems it states that the sum of forces acting on a body is zero. (or Newton's second law states that the sum of applied forces is equal to the sum of opposing forces on a body).



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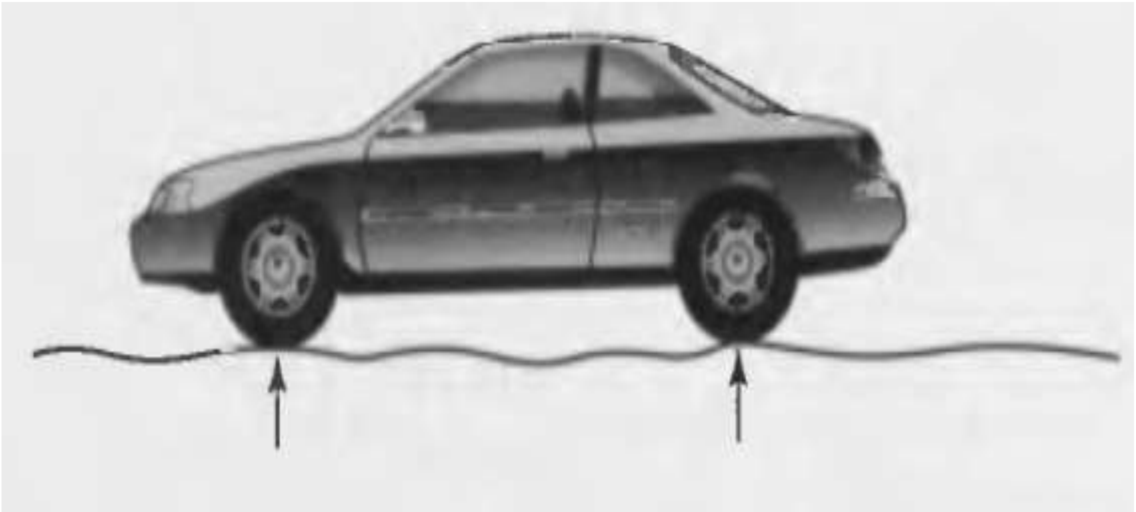
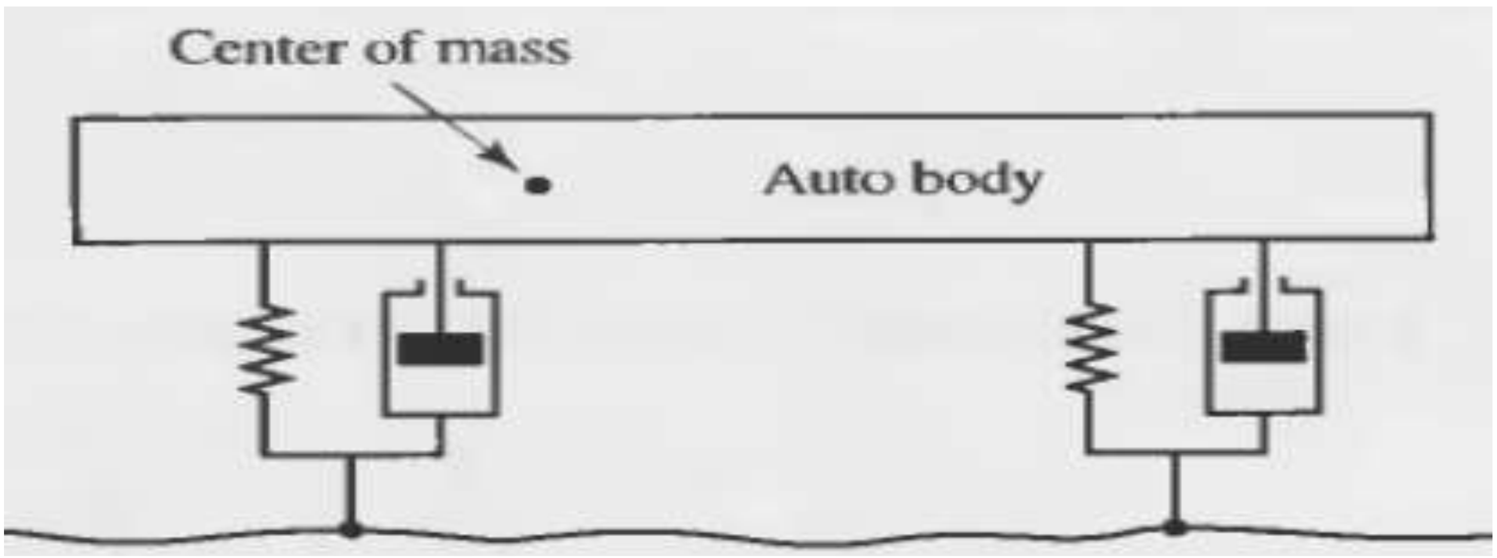
Automobile Suspension

Automobile Suspension





List of symbols used in Mech Translational System

x = Displacement, m

$v = \frac{dx}{dt}$ = Velocity, m/sec

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ = Acceleration, m/sec^2

f = Applied force, N (Newtons)

f_m = Opposing force offered by mass of the body, N

f_k = Opposing force offered by the elasticity of the body (spring), N

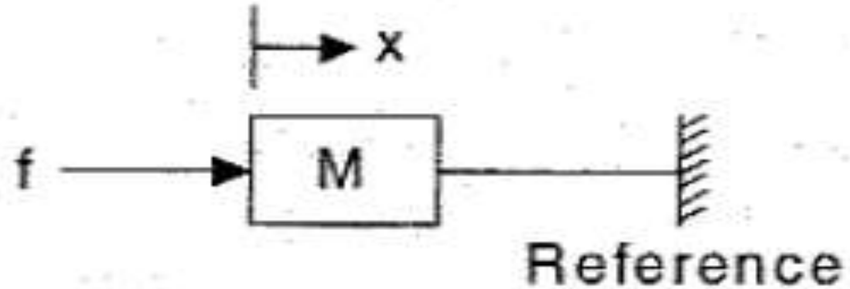
f_b = Opposing force offered by the friction of the body (dash - pot), N

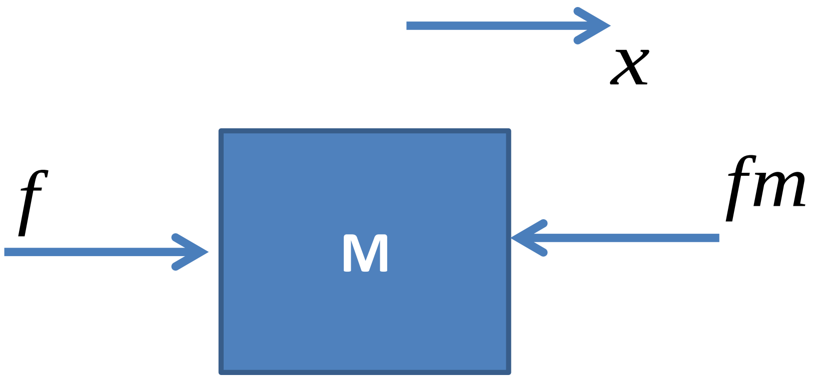
M = Mass, kg

K = Stiffness of spring, N/m

B = Viscous friction co-efficient, N-sec/m

Force Balance Equations of Idealized Elements





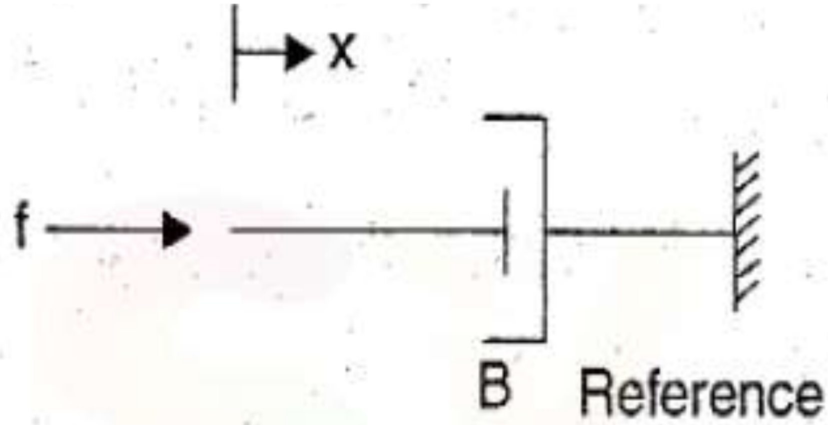
Let, f = Applied force

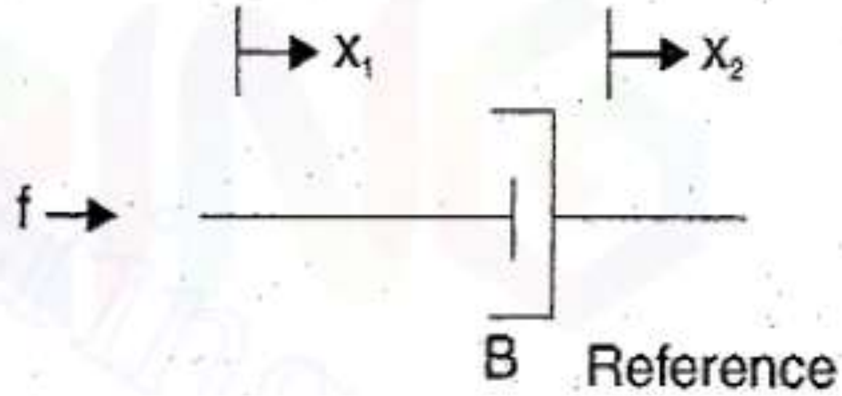
f_m = Opposing force due to mass

Here, $f_m \propto \dfrac{d^2x}{dt^2}$ or $f_m = M\dfrac{d^2x}{dt^2}$

By Newton's second law, $\boxed{f = f_m = M\dfrac{d^2x}{dt^2}}$

Force Balance Equations of Idealized Elements





$$\therefore \boxed{f = f_b = B\frac{d}{dt}(x_1 - x_2)}$$

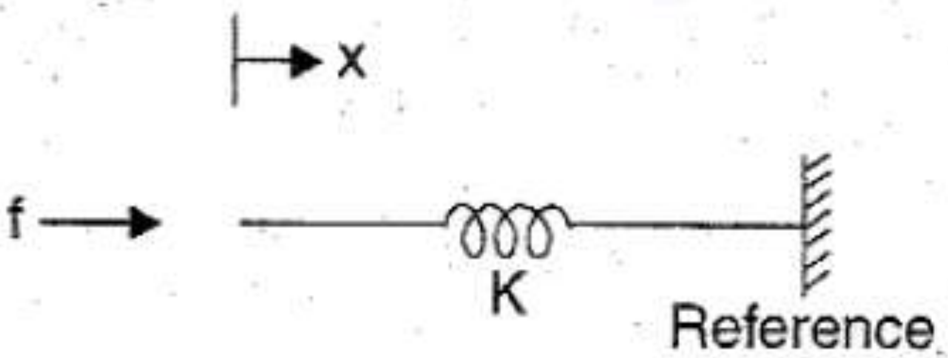
Let, f = Applied force

f_b = Opposing force due to friction

Here, $f_b \propto \frac{dx}{dt}$ or $f_b = B\frac{dx}{dt}$

By Newton's second law, $\boxed{f = f_b = B\frac{dx}{dt}}$

Force Balance Equations of Idealized Elements

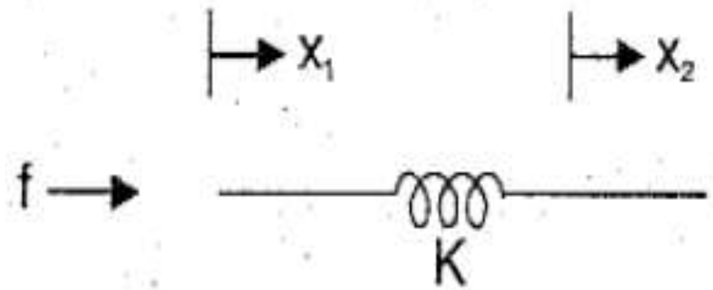


Let, f = Applied force

f_k = Opposing force due to elasticity

Here $f_k \propto x$ or $f_k = K x$

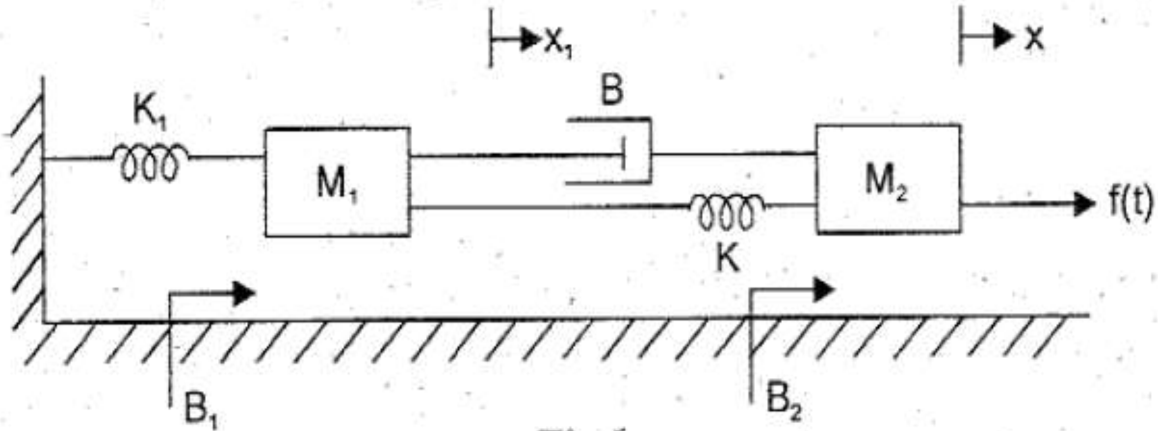
By Newton's second law, $\boxed{f = f_k = Kx}$



$$f_k \propto (x_1 - x_2)$$

$$\therefore \boxed{f = f_k = K(x_1 - x_2)}$$

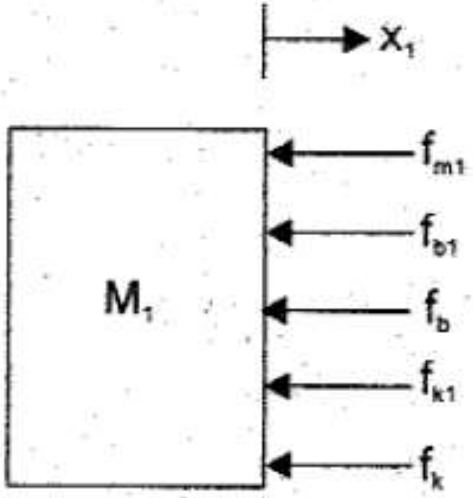
Ex.1. Write the differential equation governing the mechanical system shown in Figure and determine the transfer function.



transfer function is $\frac{X(s)}{F(s)}$

Solution:

Free body diagram fc Mass M1:



$$f_{m1} = M_1 \frac{d^2x_1}{dt^2}; \quad f_{b1} = B_1 \frac{dx_1}{dt}; \quad f_{k1} = K_1 x_1;$$

$$f_b = B\frac{d}{dt}(x_1 - x); \quad f_k = K(x_1 - x)$$

By Newton's second law,

$$f_{m1} + f_{b1} + f_b + f_{k1} + f_k = 0$$

$$\therefore M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + K_1 x_1 + K(x_1 - x) = 0$$

Apply L.T on force equation

$$\therefore M_1\frac{d^2x_1}{dt^2}+B_1\frac{dx_1}{dt}+B\frac{d}{dt}(x_1-x)+K_1x_1+K(x_1-x)=0$$

$$M_1s^2X_1(s)+B_1sX_1(s)+Bs\,[X_1(s)-X(s)]+K_1X_1(s)+K\,[X_1(s)-X(s)]=0$$

$$X_1(s)\,[M_1s^2+(B_1+B)s+(K_1+K)]-X(s)\,[Bs+K]=0$$

$$X_1(s)\,[M_1s^2+(B_1+B)s+(K_1+K)]=X(s)\,[Bs+K]$$

$$\therefore X_1(s)=X(s)\,\frac{Bs+K}{M_1s^2+(B_1+B)\,s+(K_1+K)}$$

→ **[1]**

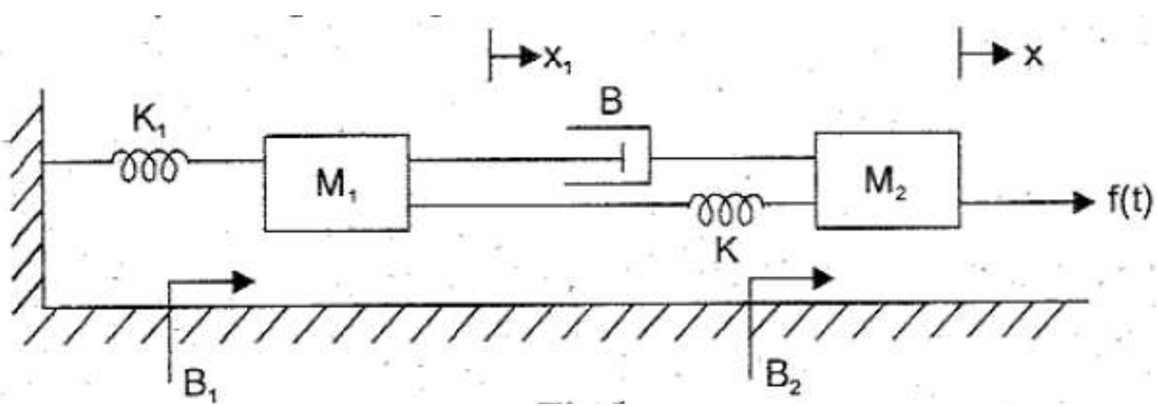
We know that:

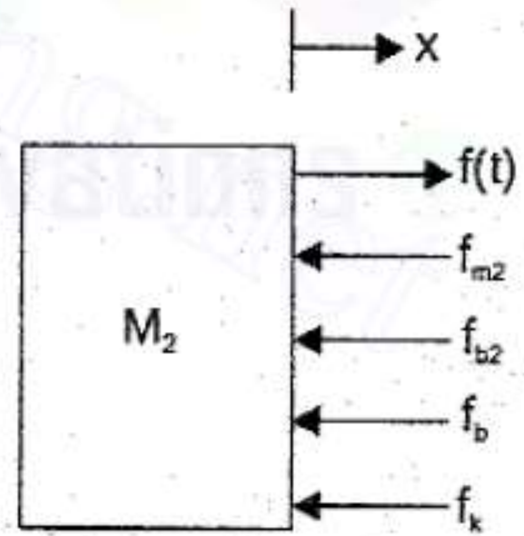
$$\mathcal{L}\{x(t)\}=X(s)$$

$$\mathcal{L}\left\{\frac{d}{dt}x(t)\right\}=s\,X(s)\ ($$

$$\mathcal{L}\left\{\frac{d^2}{dt^2}x(t)\right\}=s^2\,X(s)$$

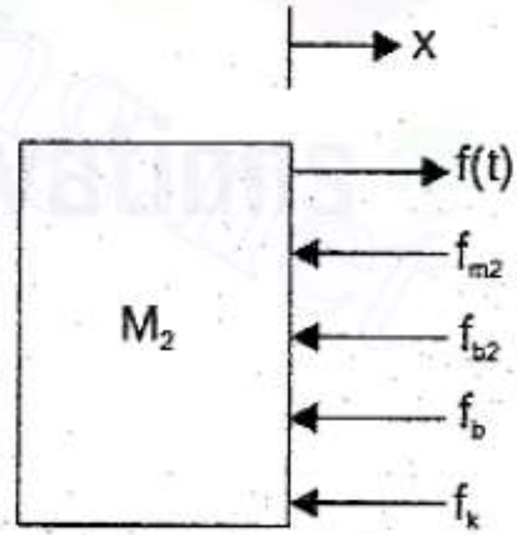
Free body diagram for Mass M2:





$$f_{m2} = M_2 \frac{d^2x}{dt^2} \quad ; \quad f_{b2} = B_2 \frac{dx}{dt}$$

$$f_b = B\frac{d}{dt}(x - x_1) \quad ; \quad f_k = K(x - x_1)$$



By Newton's second law,

$$f_{m2} + f_{b2} + f_b + f_k = f(t)$$

$$M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$M_2s^2X(s) + B_2sX(s) + Bs[X(s) - X_1(s)] + K[X(s) - X_1(s)] = F(s)$$

$$X(s)\,[M_2s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] = F(s)$$ → [2]

$$X(s)\,[M_2s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] = F(s)$$ → [2]

Substituting for $X_1(s)$ from equation (1) in equation (2) we get,

$$X(s)\,[M_2s^2 + (B_2 + B)s + K] - X(s)\,\frac{(Bs + K)^2}{M_1s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$X(s)\left[\frac{[M_2s^2 + (B_2 + B)s + K]\,[M_1s^2 + (B_1 + B)s + (K_1 + K)] - (Bs + K)^2}{M_1s^2 + (B_1 + B)\,s + (K_1 + K)}\right] = F(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{M_1s^2 + (B_1 + B)s + (K_1 + K)}{[M_1s^2 + (B_1 + B)s + (K_1 + K)]\,[M_2s^2 + (B_2 + B)s + K] - (Bs + K)^2}$$

RESULT

The differential equations governing the system are,

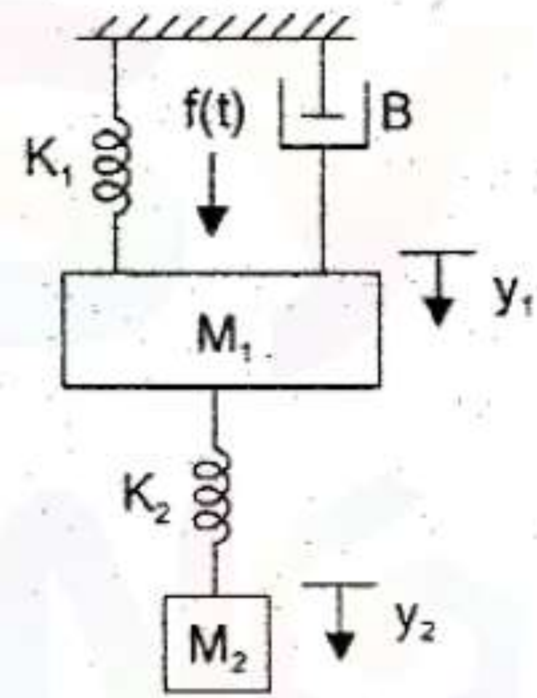
1. $M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + K_1x_1 + K(x_1 - x) = 0$

2. $M_2\frac{d^2x}{dt^2} + B_2\frac{dx}{dt} + B\frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$

The transfer function of the system is,

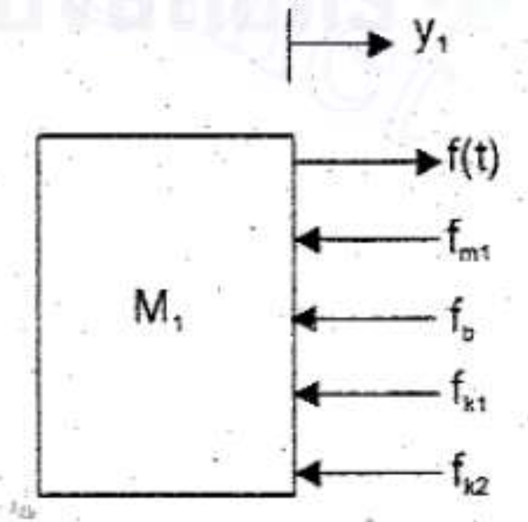
$$\frac{X(s)}{F(s)} = \frac{M_1s^2 + (B_1 + B)\,s + (K_1 + K)}{[M_1s^2 + (B_1 + B)\,s + (K_1 + K)]\,[M_2s^2 + (B_2 + B)\,s + K] - (Bs + K)^2}$$

Ex.2. Determine the transfer function $\frac{Y_2(S)}{F(S)}$ of the system shown in figure.



Solution:

Free body diagram for Mass M1:



The opposing forces are marked as f_{m1}, f_b, f_{k1} and f_{k2}

$$f_{m1} = M_1\frac{d^2y_1}{dt^2} \;;\; f_b = B\frac{dy_1}{dt} \;;\; f_{k1} = K_1y_1 \;;\; f_{k2} = K_2(y_1 - y_2)$$

By Newton's second law, $f_{m1} + f_b + f_{k1} + f_{k2} = f(t)$

$$\therefore M_1\frac{d^2y_1}{dt^2} + B\frac{dy_1}{dt} + K_1y_1 + K_2(y_1 - y_2) = f(t) \qquad .....(1)$$

$$\therefore M_1\frac{d^2y_1}{dt^2}+B\frac{dy_1}{dt}+K_1y_1+K_2(y_1-y_2)=f(t) \qquad .....(1)$$

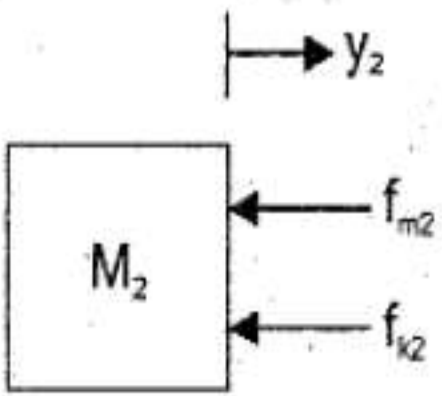
On taking Laplace transform of equation (1) with zero initial condition we get,

$$M_1s^2Y_1(s)+BsY_1(s)+K_1Y_1(s)+K_2[Y_1(s)-Y_2(s)]=F(s)$$

$$Y_1(s)[M_1s^2+Bs+(K_1+K_2)]-Y_2(s)K_2=F(s)$$

→ [2]

Free body diagram for Mass M2:



$$f_{m2}=M_2\frac{d^2y_2}{dt^2}\ ; \qquad f_{k2}=K_2(y_2-y_1)$$

By Newton's second law, $f_{m2}+f_{k2}=0$

$$\therefore M_2\frac{d^2y_2}{dt^2}+K_2(y_2-y_1)=0$$

$$\therefore M_2 \frac{d^2 y_2}{dt^2} + K_2(y_2 - y_1) = 0$$

On taking Laplace transform of above equation we get,

$$M_2 s^2 Y_2(s) + K_2[Y_2(s) - Y_1(s)] = 0$$

$$Y_2(s)\,[M_2 s^2 + K_2] - Y_1(s)\,K_2 = 0$$

$$\therefore Y_1(s) = Y_2(s)\,\frac{M_2 s^2 + K_2}{K_2}$$ → [3]

Substituting for $Y_1(s)$ from equation (3) in equation (2) we get,

$$Y_2(s)\left[\frac{M_2 s^2 + K_2}{K_2}\right]\left[M_1 s^2 + Bs + (K_1 + K_2)\right] - Y_2(s)\,K_2 = F(s)$$

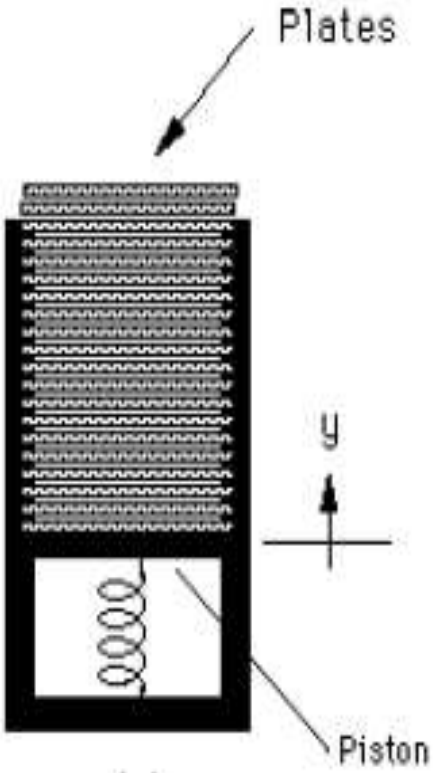
$$Y_2(s)\left[\frac{(M_2 s^2 + K_2)\,[M_1 s^2 + Bs + (K_1 + K_2)] - K_2^2}{K_2}\right] = F(s)$$

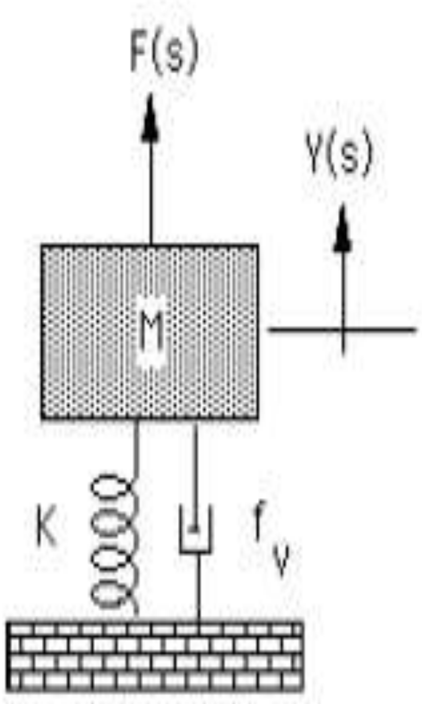
Transfer Function:

$$\therefore \frac{Y_2(s)}{F(s)} = \frac{K_2}{\left[M_1 s^2 + Bs + (K_1 + K_2)\right]\left[M_2 s^2 + K_2\right] - K_2^2}$$

Exercise Problem-2

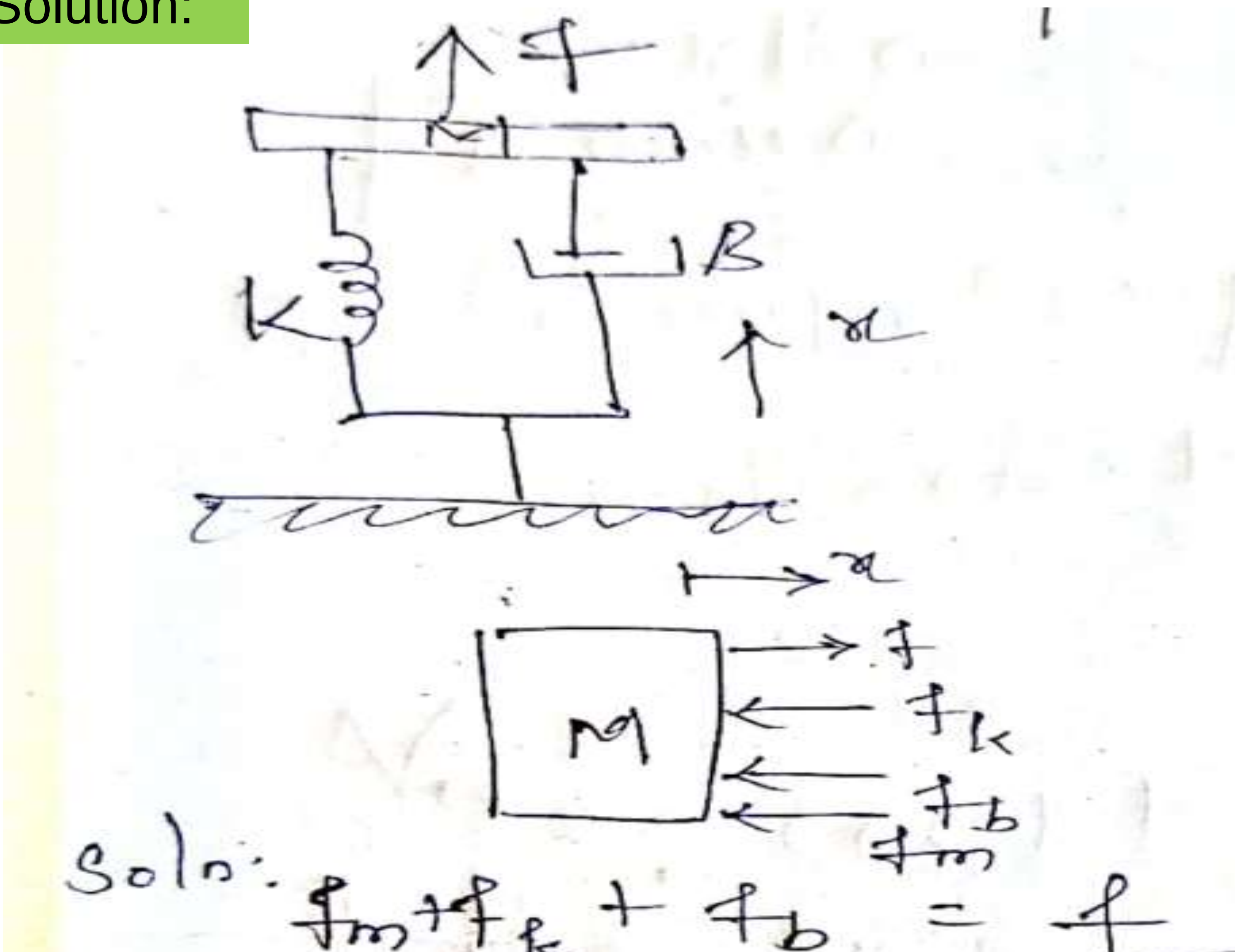
Draw the physical model and mathematical model of the Restaurant plate dispenser and thus determine the transfer function of the system





Transfer function of Restaurant plate dispenser

Solution:



Soln:

$$f_m + f_k + f_b = f$$

$$kx + B\frac{dx}{dt} + M\frac{d^2x}{dt^2} = f$$

Apply L.T

$$k\,x(s) + Bs\,x(s) + Ms^2 x(s) = F(s)$$

$$x(s)\left[k + Bs + Ms^2\right] = F(s)$$

$$\frac{x(s)}{F(s)} = \frac{1}{k + Bs + Ms^2}$$ Transfer function.

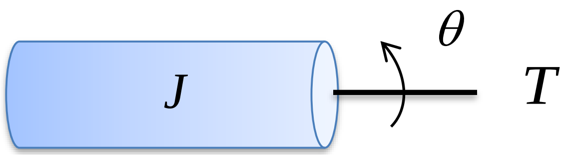
Rotational Mechanical Systems

Basic Elements

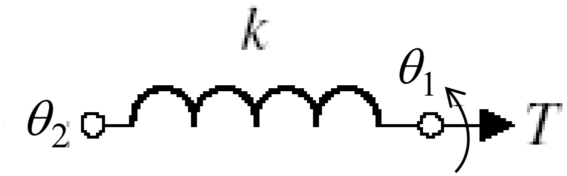
Input: T – Torque, A force which tends to cause rotation

Output: ϴ - Angular Displacement

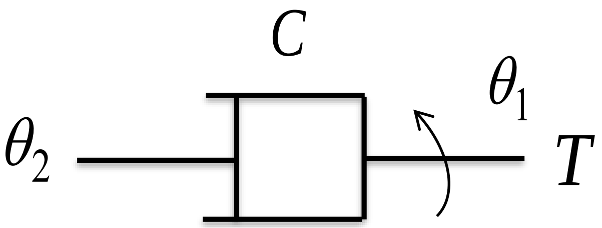
Moment of Inertia of Mass, J



Rotational Spring, K



Rotational Dash, B



List of symbols used in Mech Rotational System

θ = Angular displacement, rad

$\frac{d\theta}{dt}$ = Angular velocity, rad/sec

$\frac{d^2\theta}{dt^2}$ = Angular acceleration, rad/sec^2

T = Applied torque, N-m

J = Moment of inertia, Kg-m^2/rad

B = Rotational frictional coefficient, N-m/(rad/sec)

K = Stiffness of the spring, N-m/rad

Torque Balance Equations of Idealized Elements

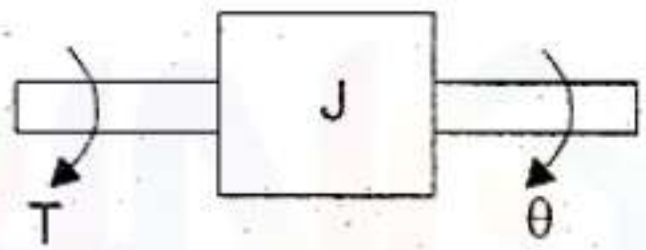


Fig 1.14 : Ideal rotational mass element.

$$T = T_j = J\frac{d^2\theta}{dt^2}$$

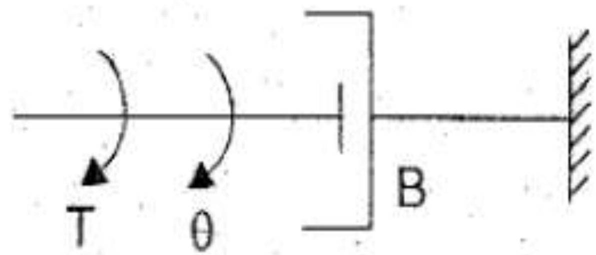


Fig 1.15 : Ideal rotational dash-pot with one end fixed to reference.

$$T = T_b = B\frac{d\theta}{dt}$$

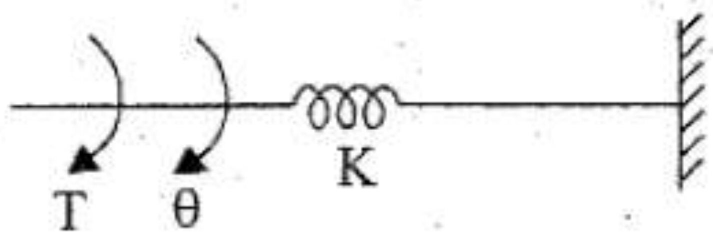
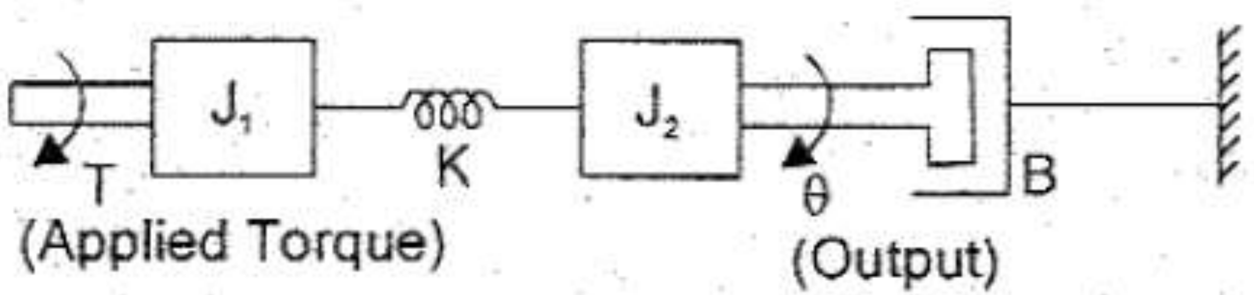


Fig 1.17 : Ideal spring with one end fixed to reference.

$$T = T_k = K\theta$$

Ex.3. Write the differential equation governing the mechanical rotational system shown in Figure and determine the transfer function



Solution:

In the given system, applied torque T is the input and angular displacement θ is the output.

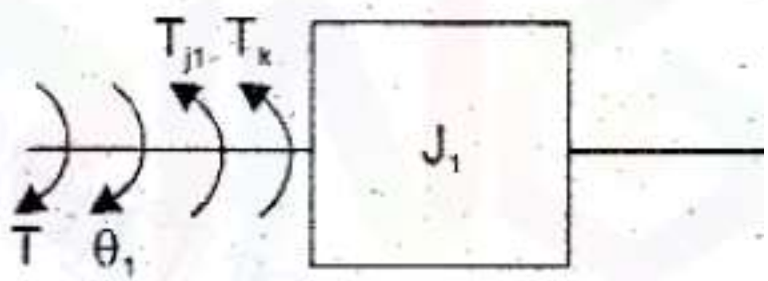
Let, Laplace transform of T $= \mathcal{L}\{T\} = T(s)$

Laplace transform of θ $= \mathcal{L}\{\theta\} = \theta(s)$

Laplace transform of $\theta_1 = \mathcal{L}\{\theta_1\} = \theta_1(s)$

Hence the required transfer function is $\dfrac{\theta(s)}{T(s)}$

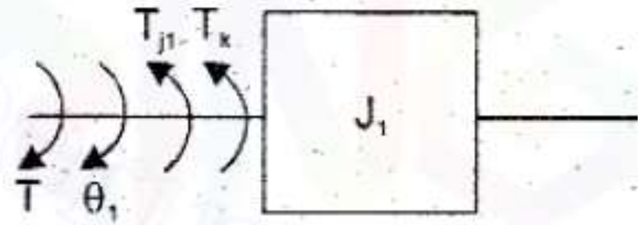
Free body diagram for Moment of Inertia J1:



$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2} \quad ; \quad T_k = K(\theta_1 - \theta)$$

By Newton's second law, $T_{j1} + T_k = T$

$$J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$



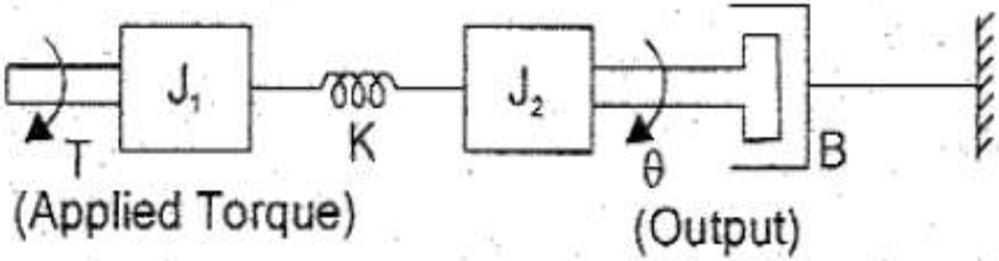
$$J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta = T \qquad .....(1)$$

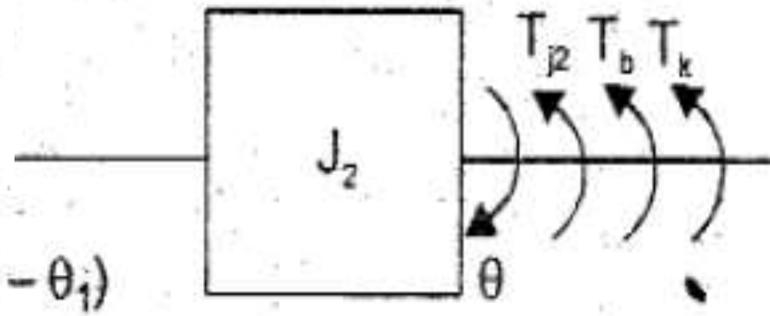
On taking Laplace transform of equation (1) with zero initial conditi

$$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$$

$$(J_1 s^2 + K)\,\theta_1(s) - K\,\theta(s) = T(s) \qquad .....(2)$$

Free body diagram for Moment





$$T_{j2} = J_2 \frac{d^2\theta}{dt^2} \quad ; \quad T_b = B\frac{d\theta}{dt} \quad ; \quad T_k = K(\theta - \theta_1)$$

By Newton's second law, $T_{j2} + T_b + T_k = 0$

$$\therefore J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta - K\theta_1 = 0 \qquad \rightarrow \text{[3]}$$

On taking Laplace transform of above equation

$$J_2 s^2\theta(s) + B\, s\, \theta(s) + K\theta(s) - K\theta_1(s) = 0$$

$$(J_2 s^2 + Bs + K)\,\theta(s) - K\theta_1(s) = 0$$

$$\theta_1(s) = \frac{(J_2 s^2 + Bs + K)}{K}\,\theta(s)$$ → [4]

Substituting for $\theta_1(s)$ from equation (3) in equation (2) we get,

$$(J_1 s^2 + K)\,\frac{(J_2 s^2 + Bs + K)}{K}\,\theta(s) - K\theta(s) = T(s)$$

$$\left[\frac{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}{K}\right]\theta(s) = T(s)$$

Transfer Function:

$$\therefore \frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}$$

Ex.4. Write the differential equation governing the mechanical rotational system shown in Figure and determine the transfer function.

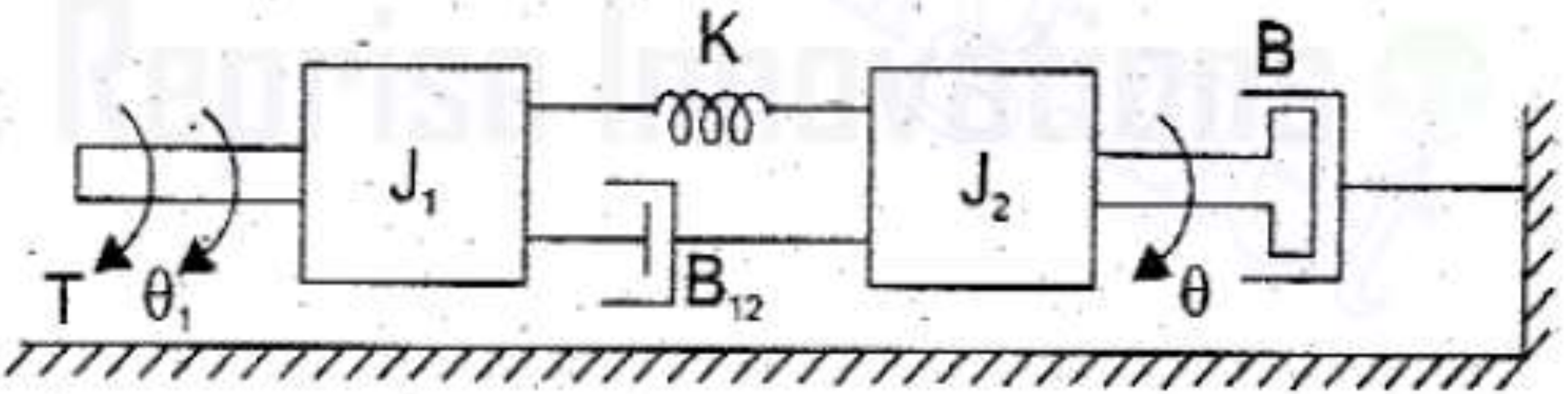
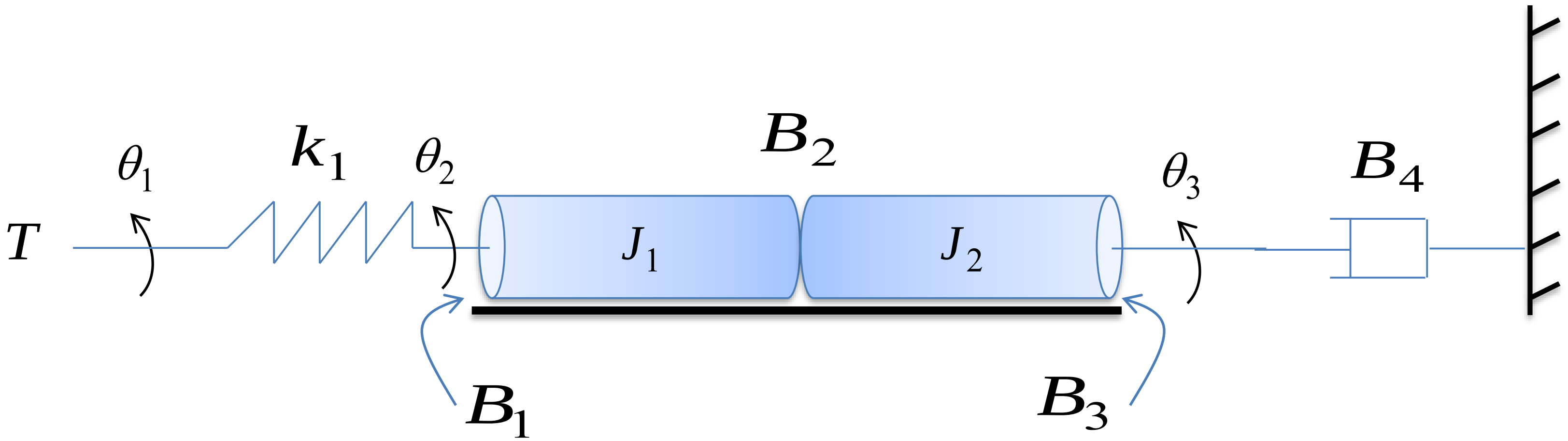
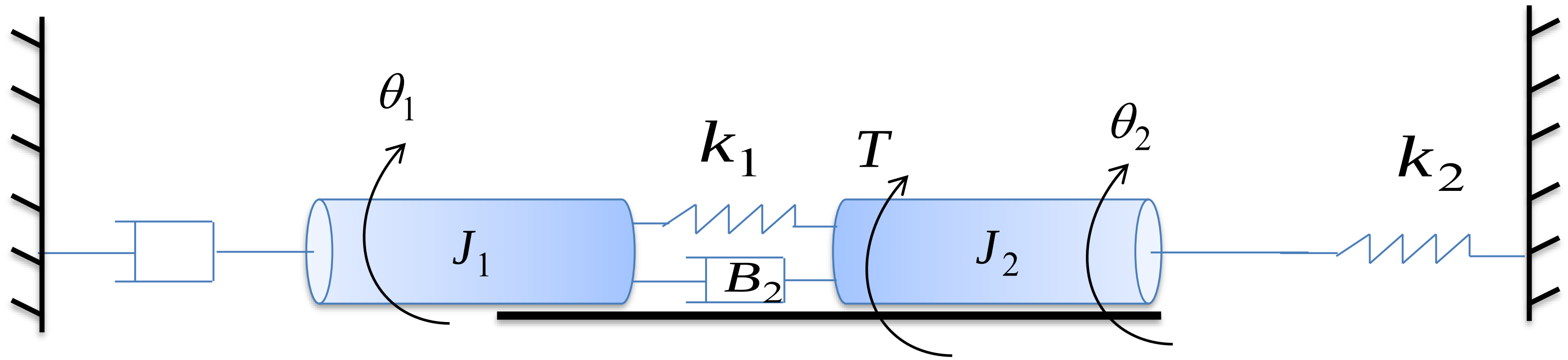


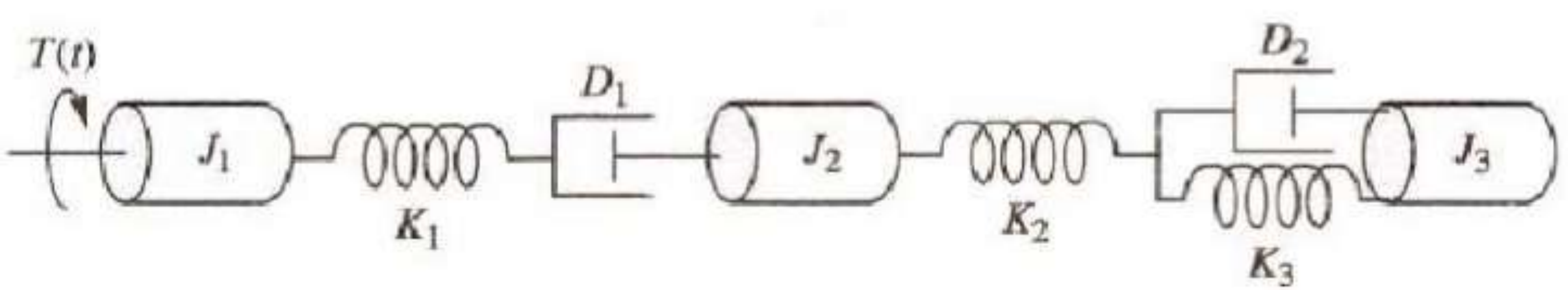
Fig 1.



T
θ1
k1
θ2
B2
J1
J2
θ3
B4
B1
B3



θ1
k1
T
θ2
k2
J1
B2
J2



T(t)
J1
K1
D1
J2
K2
D2
K3
J3

Ex.4. Write the differential equation governing the mechanical rotational system shown in Figure and determine the transfer function.

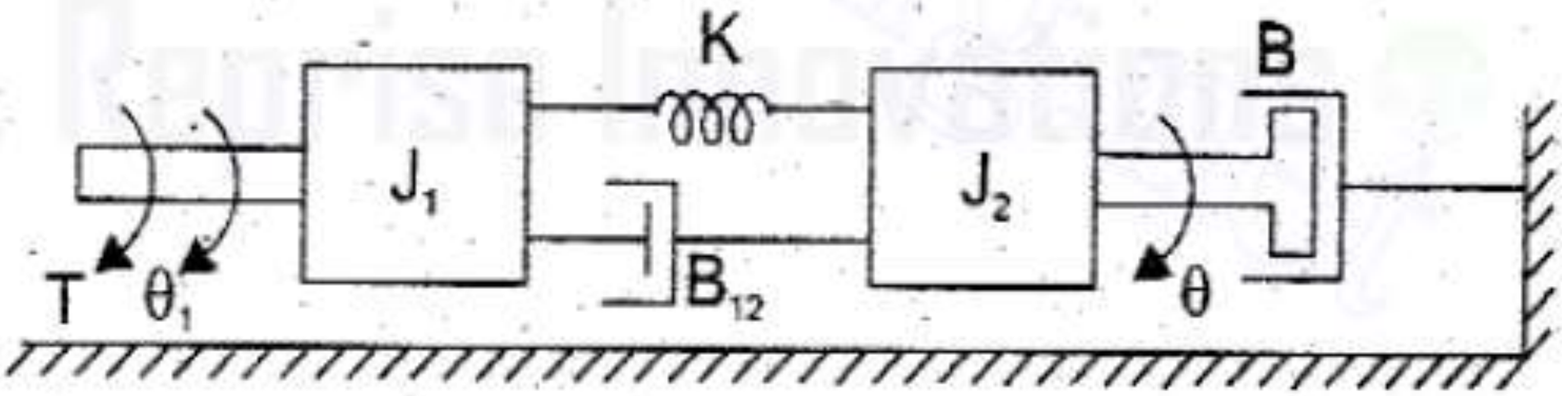
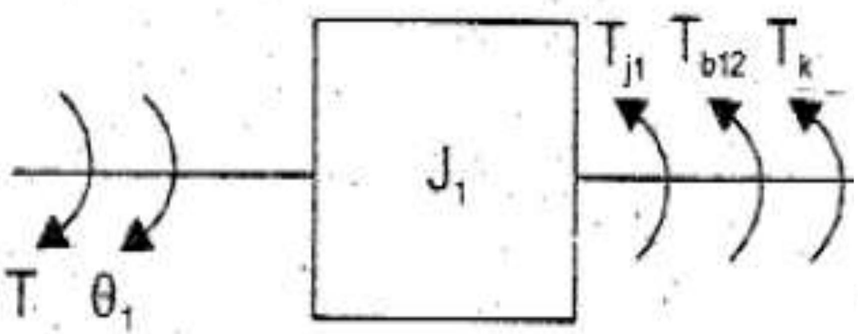


Fig 1.

Solution:

$$T_k = K(\theta_1 - \theta)$$



$$T_{j1} = J_1 \frac{d^2\theta_1}{dt^2} \quad ; \quad T_{b12} = B_{12}\frac{d}{dt}(\theta_1 - \theta)$$

By Newton's second law, $T_{j1} + T_{b12} + T_k = T$

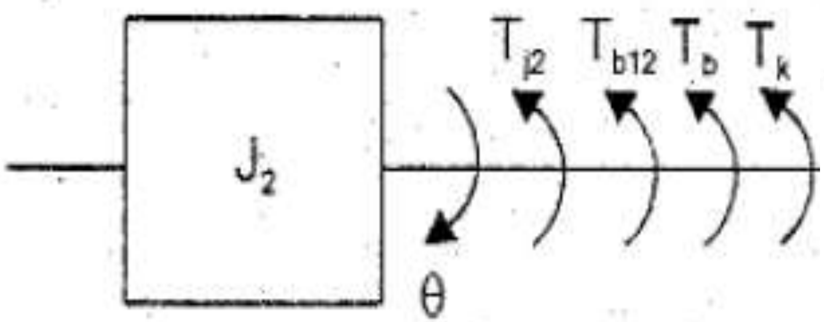
$$J_1\frac{d^2\theta_1}{dt^2}+B_{12}\frac{d}{dt}(\theta_1-\theta)+K(\theta_1-\theta)=T$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$J_1s^2\theta_1(s)+s\,B_{12}\,[\theta_1(s)-\theta(s)]+K\theta_1(s)-K\theta(s)=T(s)$$

$$\theta_1(s)\,[J_1s^2+sB_{12}+K]-\theta(s)\,[sB_{12}+K]=T(s)$$

The free body diagram of mass with moment of inertia J_2 is shown



$$T_{j2}=J_2\frac{d^2\theta}{dt^2}\quad;\quad T_{b12}=B_{12}\frac{d}{dt}(\theta-\theta_1)$$

$$T_b=B\frac{d\theta}{dt}\quad;\quad T_k=K(\theta-\theta_1)$$

By Newton's second law, $T_{j2}+T_{b12}+T_b+T_k=0$

$$J_2 \frac{d^2\theta}{dt^2} + B_{12} \frac{d}{dt}(\theta - \theta_1) + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

$$J_2 \frac{d^2\theta}{dt^2} - B_{12} \frac{d\theta_1}{dt} + \frac{d\theta}{dt}(B_{12} + B) + K\theta - K\theta_1 = 0$$

On taking Laplace transform of above equation with zero initial conditions we get,

$$J_2 s^2\theta(s) - B_{12} s\theta_1(s) + s\theta(s)\,[B_{12} + B] + K\theta(s) - K\theta_1(s) = 0$$

$$\theta(s)\,[s^2 J_2 + s(B_{12} + B) + K] - \theta_1(s)\,[sB_{12} + K] = 0$$

$$\theta_1(s) = \frac{[s^2 J_2 + s(B_{12} + B) + K]}{[sB_{12} + K]}\,\theta(s)$$

Substituting for $\theta_1(s)$ from equation (2) in equation (1) we get,

$$[J_1s^2 + sB_{12} + K]\ \frac{[J_2s^2 + s(B_{12} + B) + K]\ \theta(s)}{(sB_{12} + K)} - (sB_{12} + K)\ \theta(s) = T(s)$$

$$\left[\frac{(J_1s^2 + sB_{12} + K)\ [J_2s^2 + s(B_{12} + B) + K] - (sB_{12} + K)^2}{(sB_{12} + K)}\right]\theta(s) = T(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{(sB_{12} + K)}{(J_1s^2 + sB_{12} + K)\ [J_2s^2 + s(B_{12} + B) + K] - (sB_{12} + K)^2}$$

Block Diagram Reduction Techniques

Introduction

- Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

The simplest form of the block diagram is the single block, with one input and one output.

The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the output.

$$G(s) = R(s)/C(s)$$

Need for block diagram reduction

- It is normally required to reduce multiple blocks into single block or for convenient understanding it may sometimes required to rearrange the blocks from its original order.

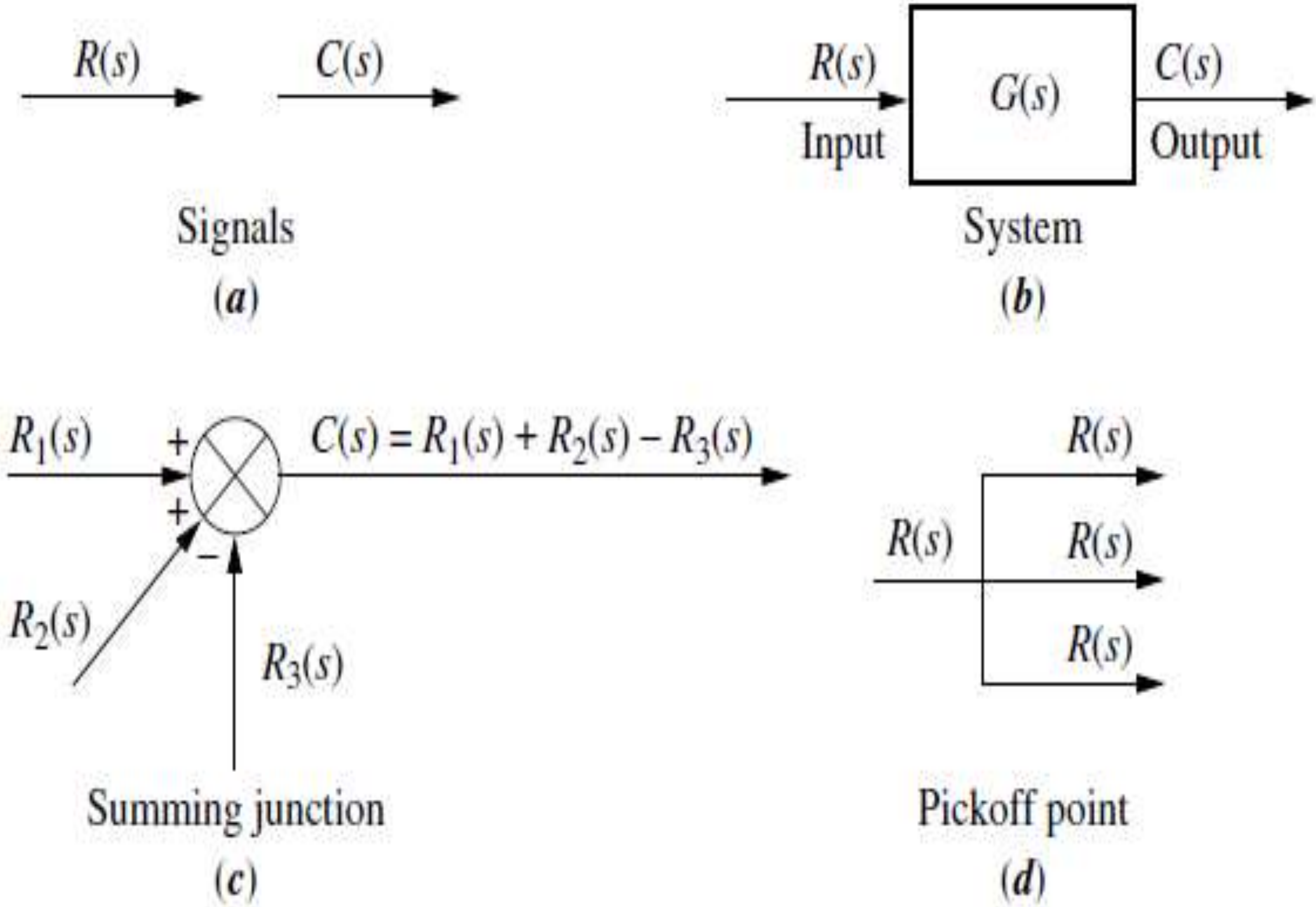
- For the calculation of Transfer function its required to be reduced.

Components of a Block Diagram

- System components are alternatively called elements of the system.

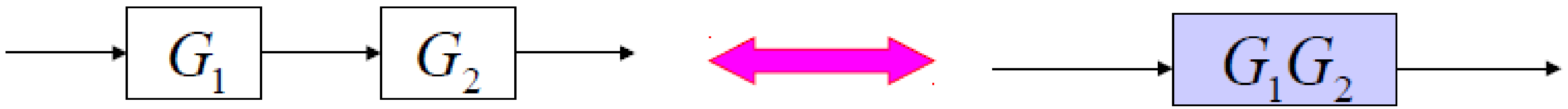
- Block diagram has four components:

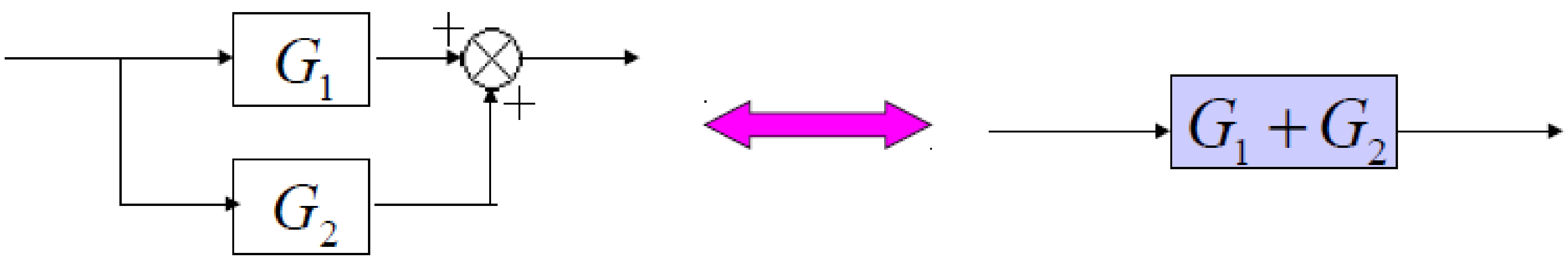
  - ***Signals***
  - ***System/ block***
  - ***Summing junction***
  - ***Pick-off/ Take-off point***


R(s)
C(s)
Signals
(a)
R(s)
Input
G(s)
C(s)
Output
System
(b)
R1(s)
+
+
−
C(s) = R1(s) + R2(s) − R3(s)
R2(s)
R3(s)
Summing junction
(c)
R(s)
R(s)
R(s)
R(s)
Pickoff point
(d)

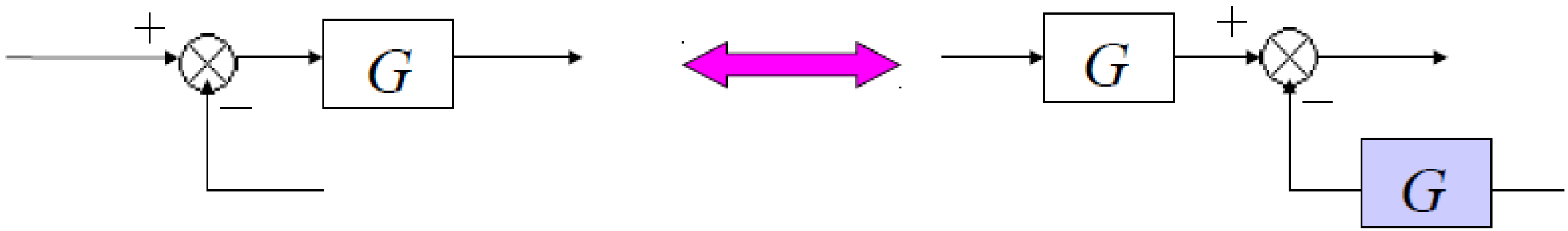
Rules for Block Diagram Reduction Techniques

1. Combining blocks which are in cascade or in parallel



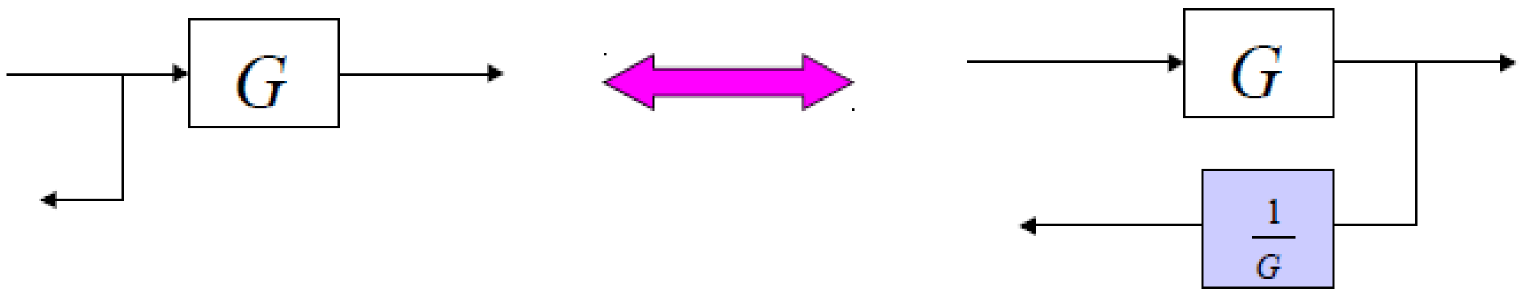


2. Moving a summing point behind a block

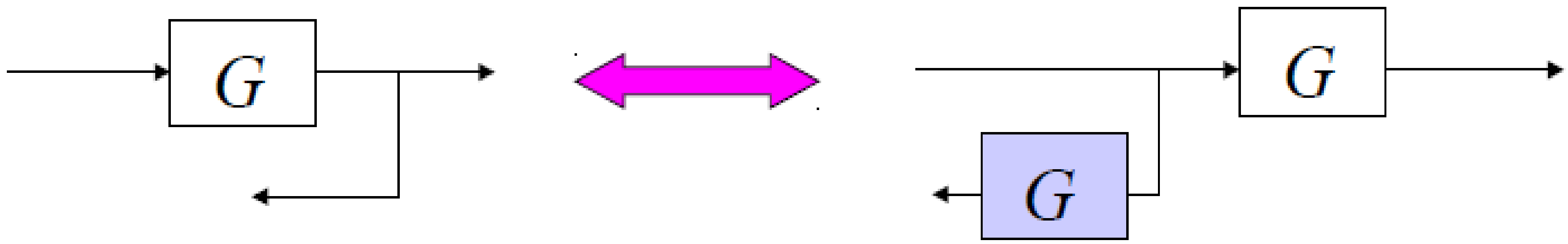


Rules for Block Diagram Reduction Techniques

4. Moving a pickoff point behind a block

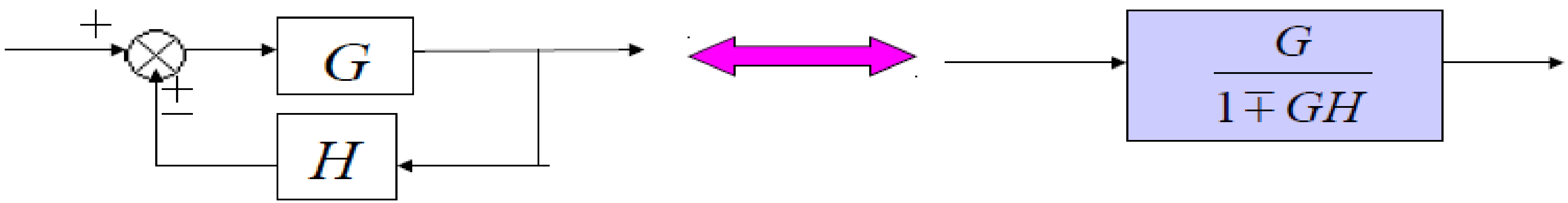


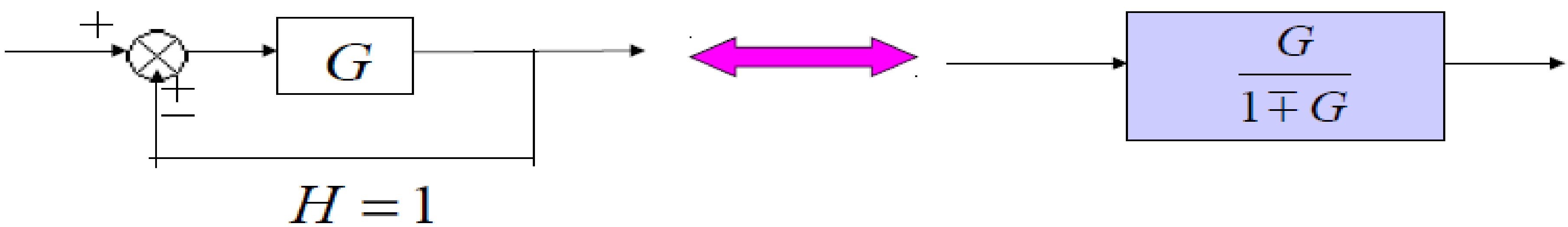
5. Moving a pickoff point ahead of a block



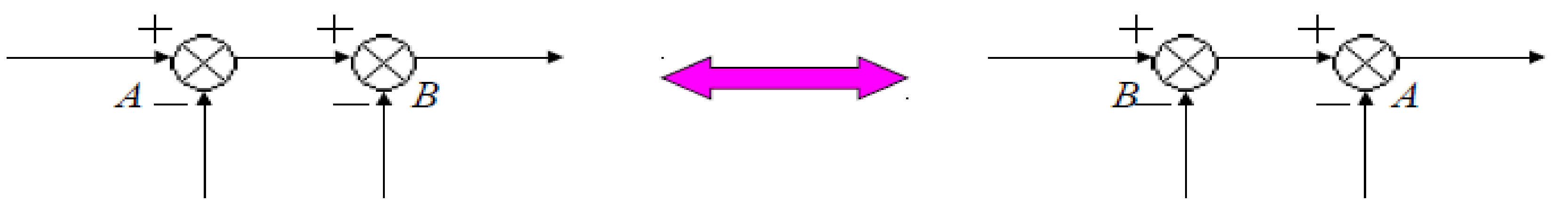
Rules for Block Diagram Reduction Techniques

6. Eliminating a feedback loop

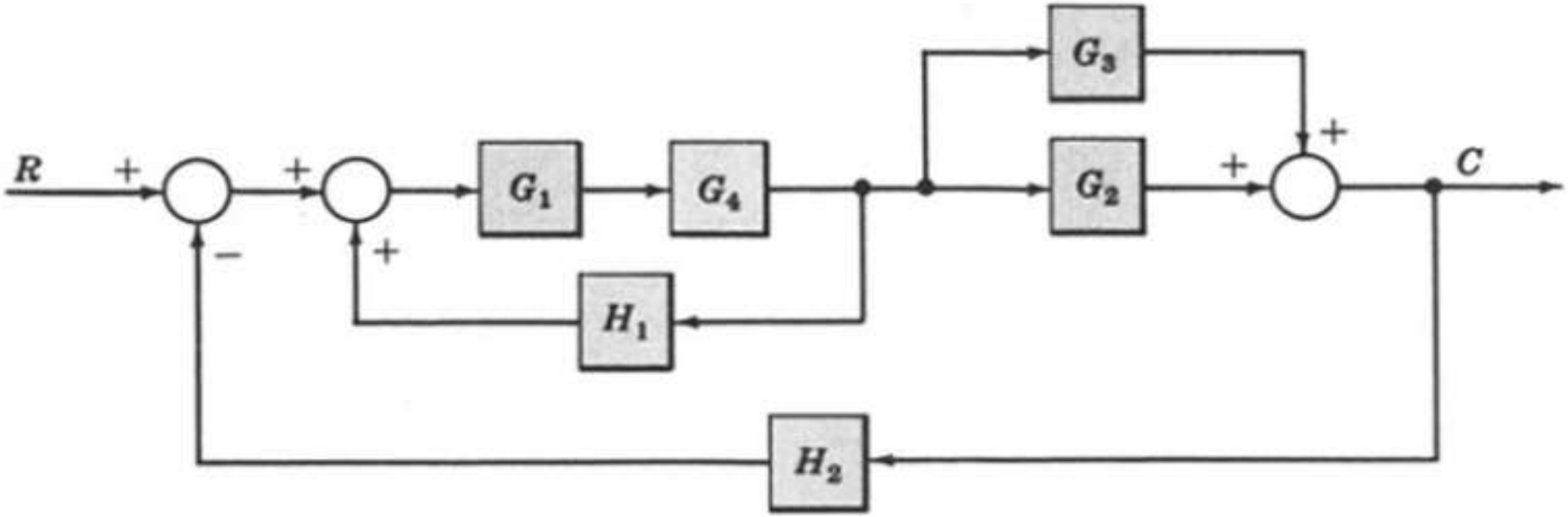




7. Swapping with two adjacent summing points



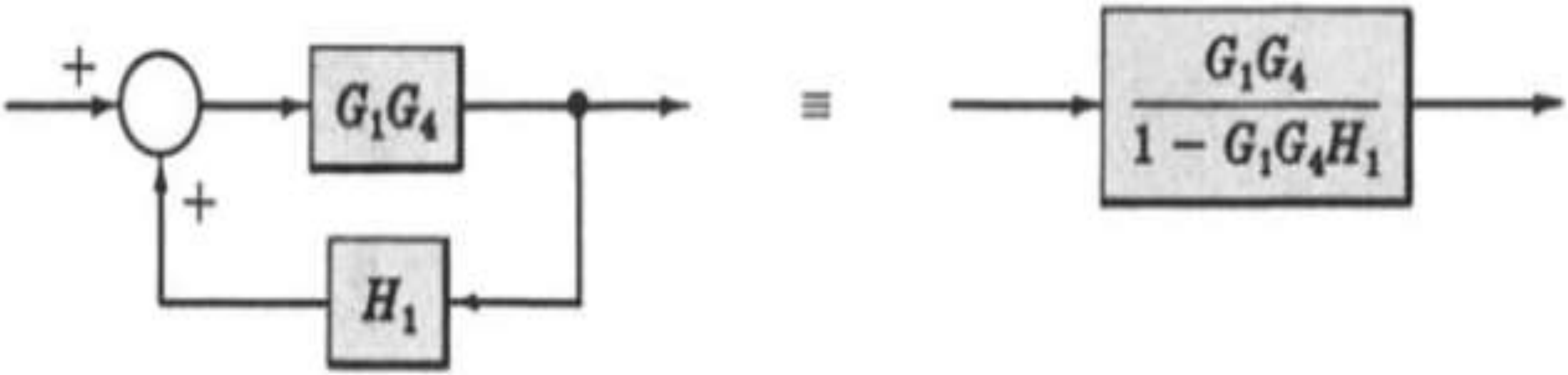
Ex.1: Reduce the Block Diagram to Canonical Form.



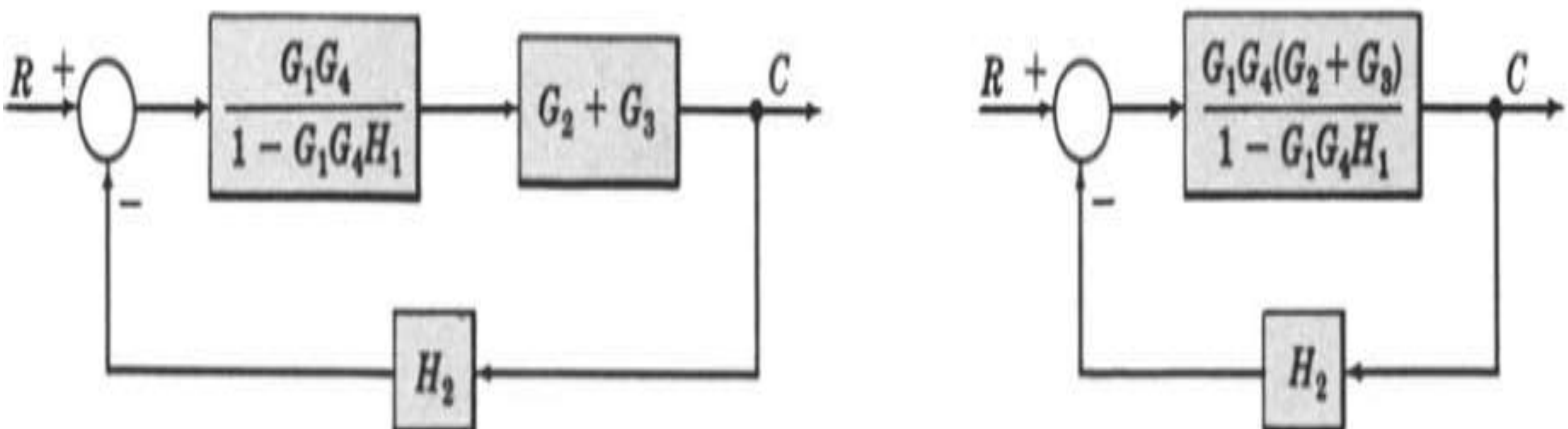
Step 1: Combine all cascade blocks using Transformation 1.

Step 2: Combine all parallel blocks using Transformation 2.

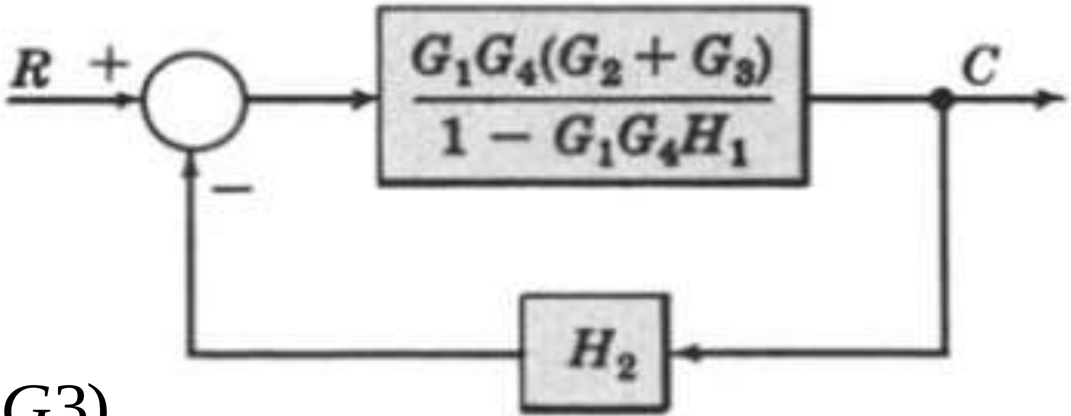
Step 3: Eliminate all minor feedback loops using Transformation 4.



Step 5: Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.



Transfer function:

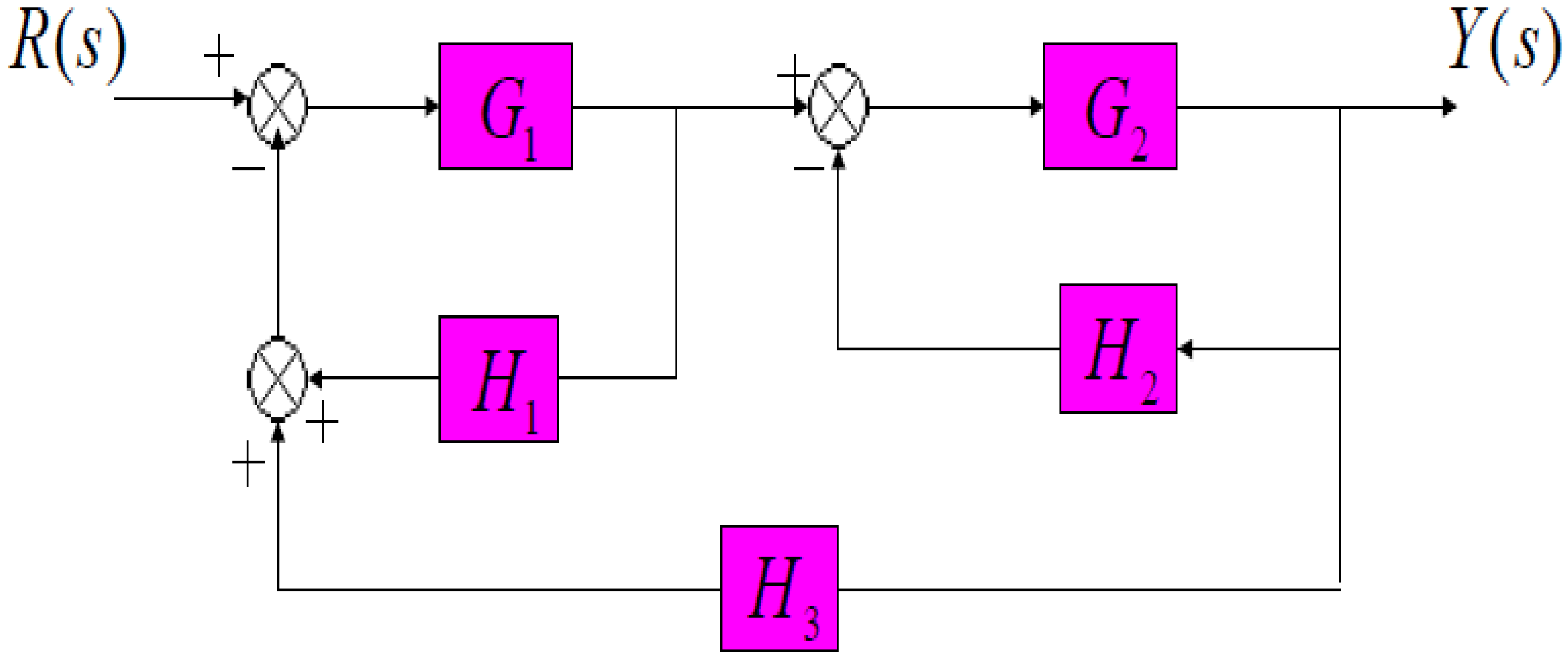


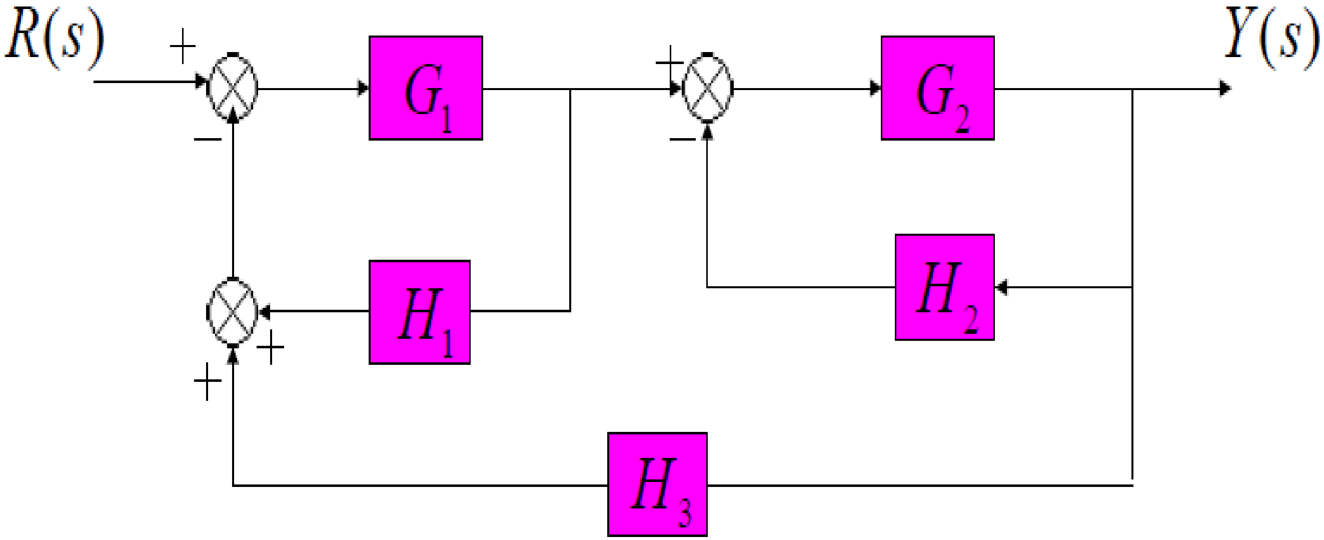
$$\frac{C(S)}{R(S)} = \frac{\dfrac{G1G4(G2+G3)}{1-G1G4H1}}{1+\left[\dfrac{G1G4(G2+G3)}{1-G1G4H1}\right]H2}$$

$$\frac{C(S)}{R(S)} = \frac{\left(\dfrac{G1G4(G2+G3)}{1-G1G4H1}\right)}{\left(\dfrac{(1-G1G4H1)+G1G4H2(G2+G3)}{1-G1G4H1}\right)}$$

$$\frac{C(S)}{R(S)} = \frac{G1G2G4+G1G3G4}{(1-G1G4H1)+G1G2G4H2+G1G3G4H2)}$$

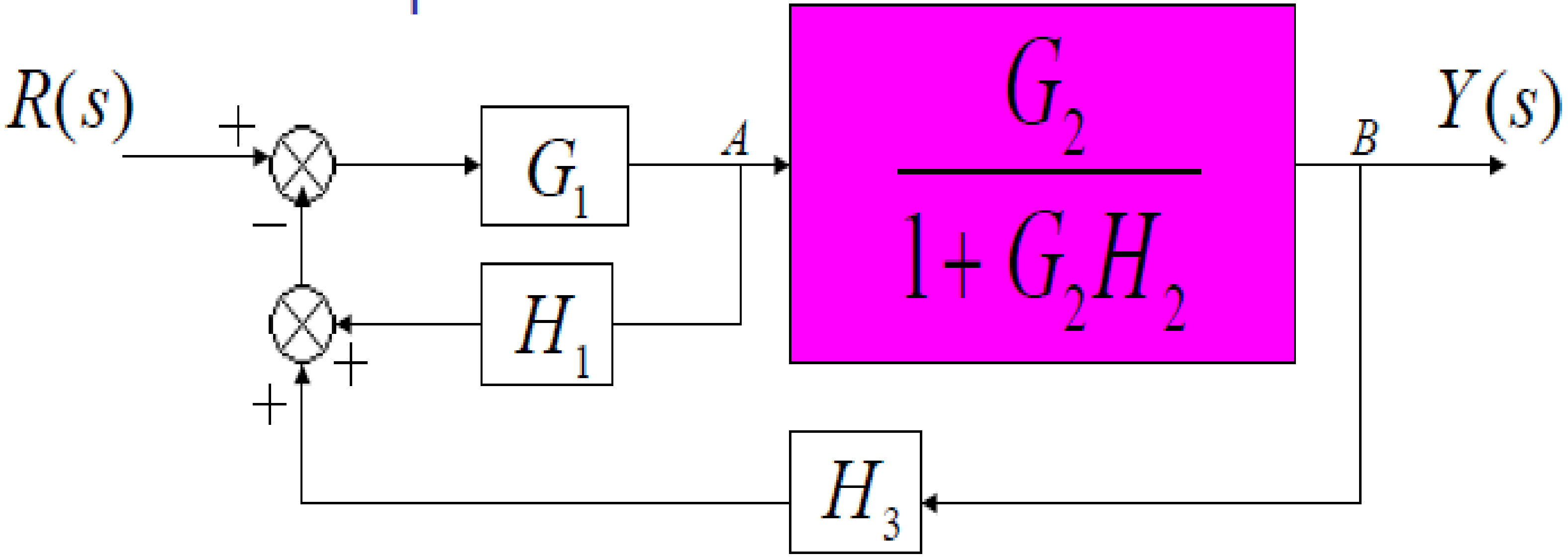
Ex. 3: Find the transfer function of the following system using block diagram reduction techniques





Solution:

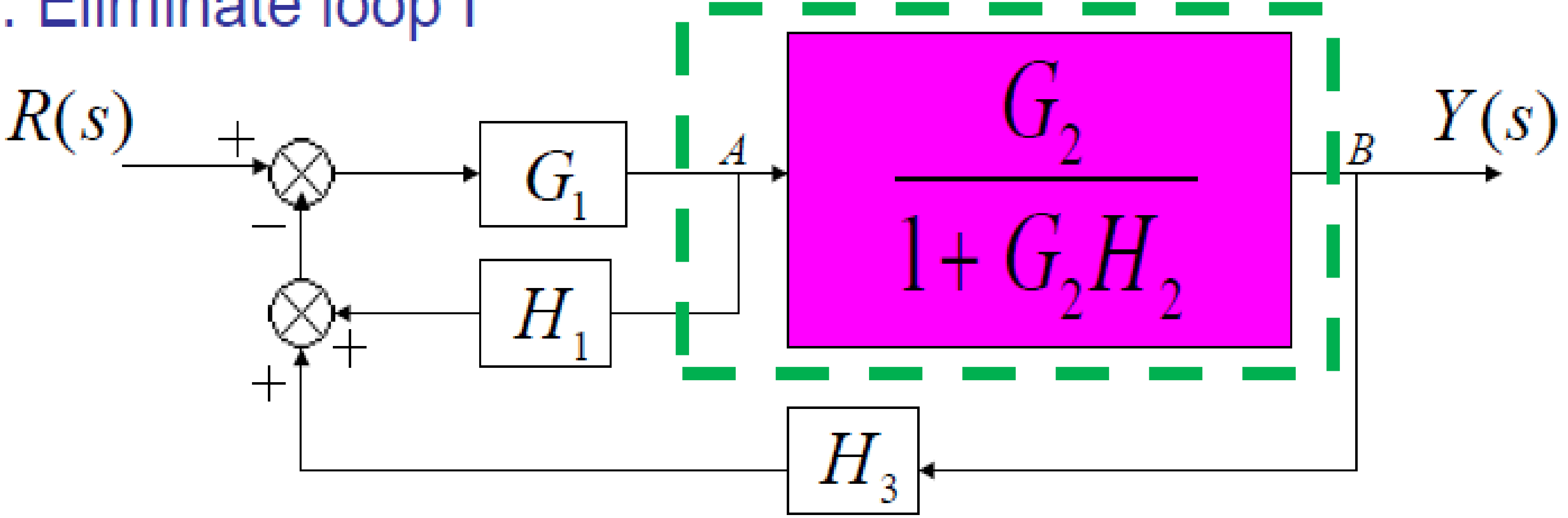
1. Eliminate loop I



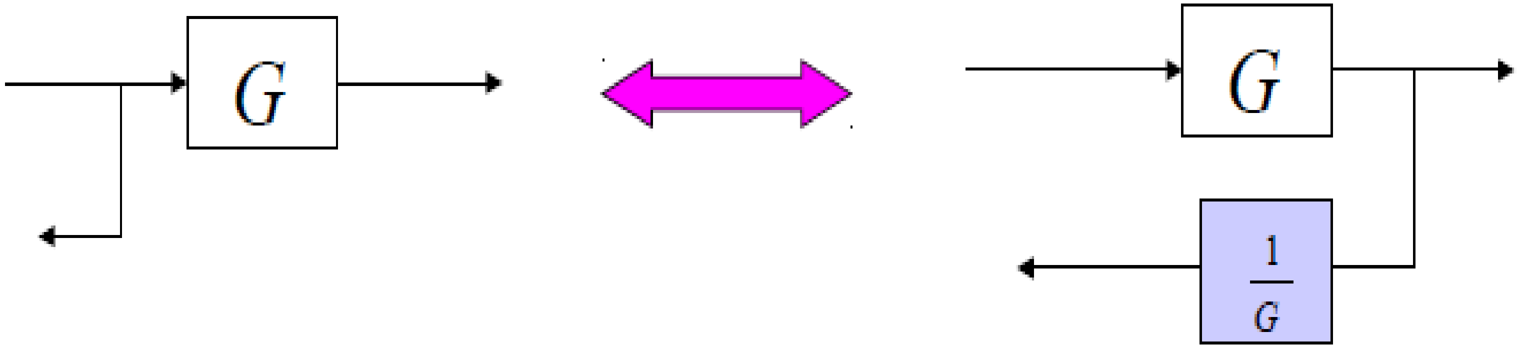
$$\frac{G_2}{1+G_2H_2}$$

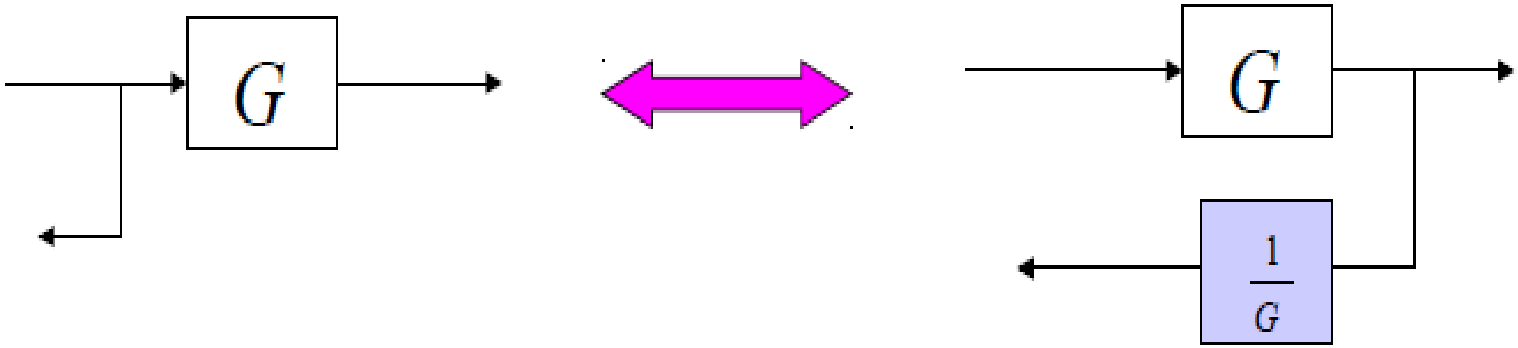
Solution:

1. Eliminate loop I

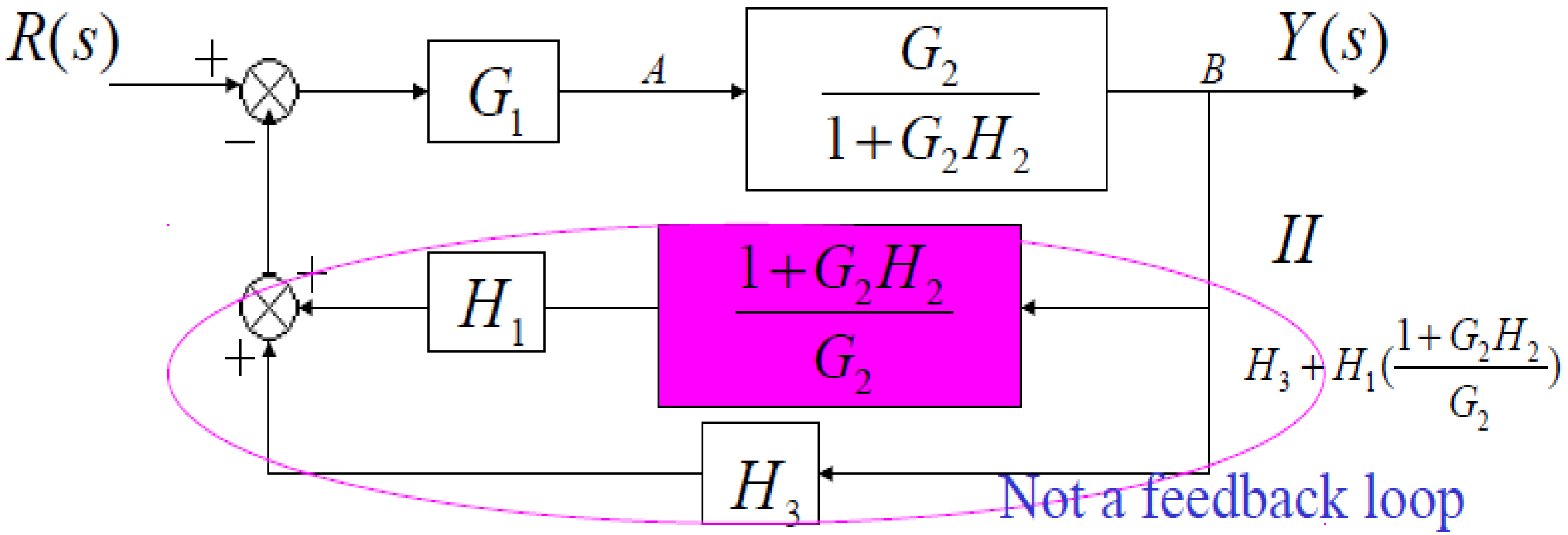


2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$

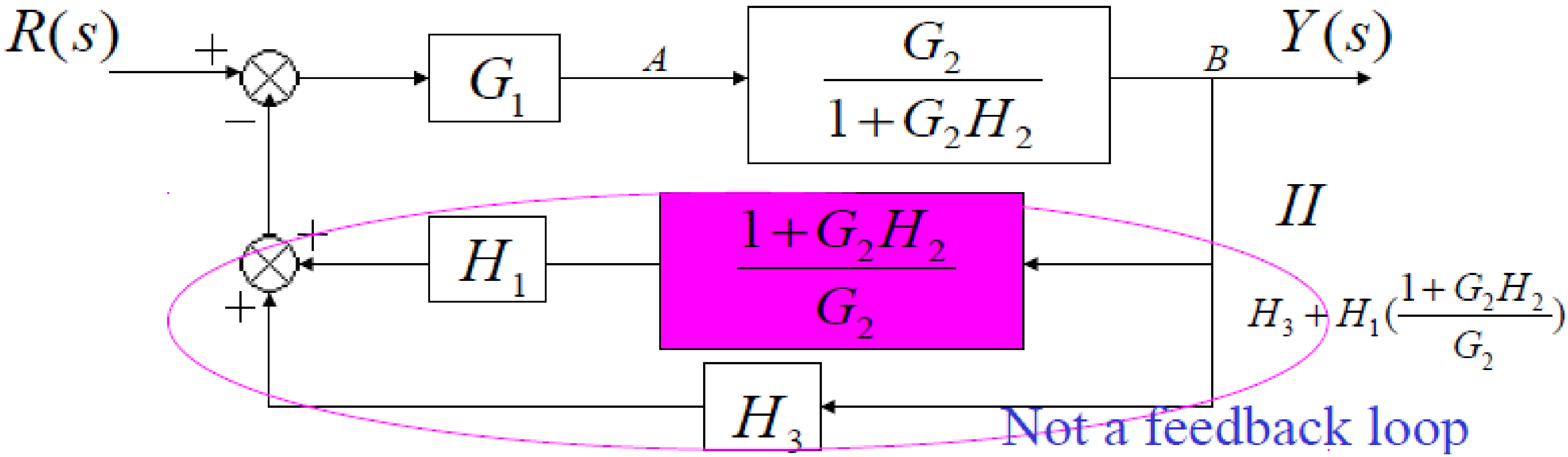




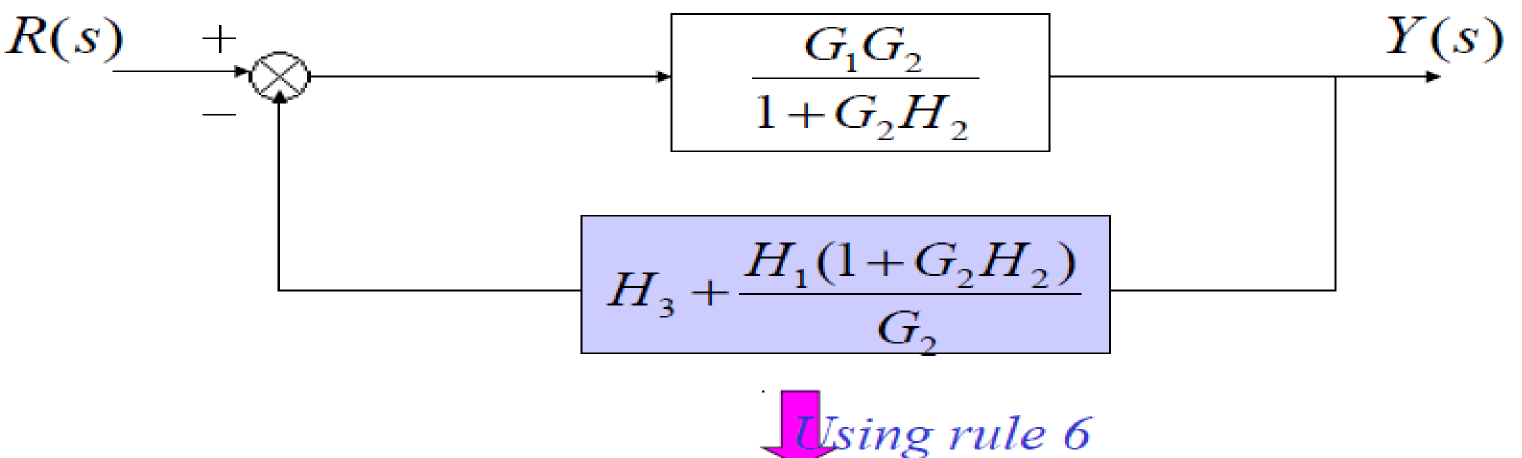
2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



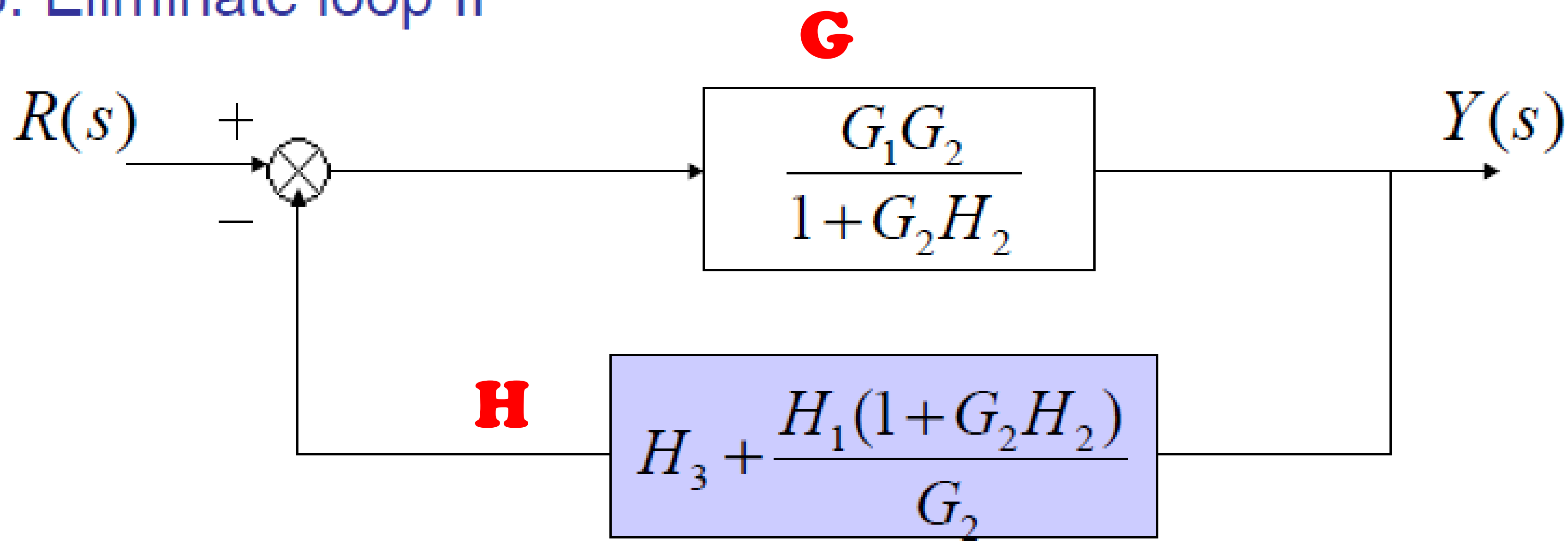
2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



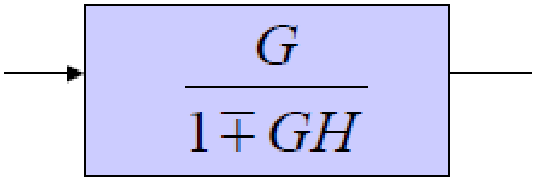
3. Eliminate loop II



3. Eliminate loop II

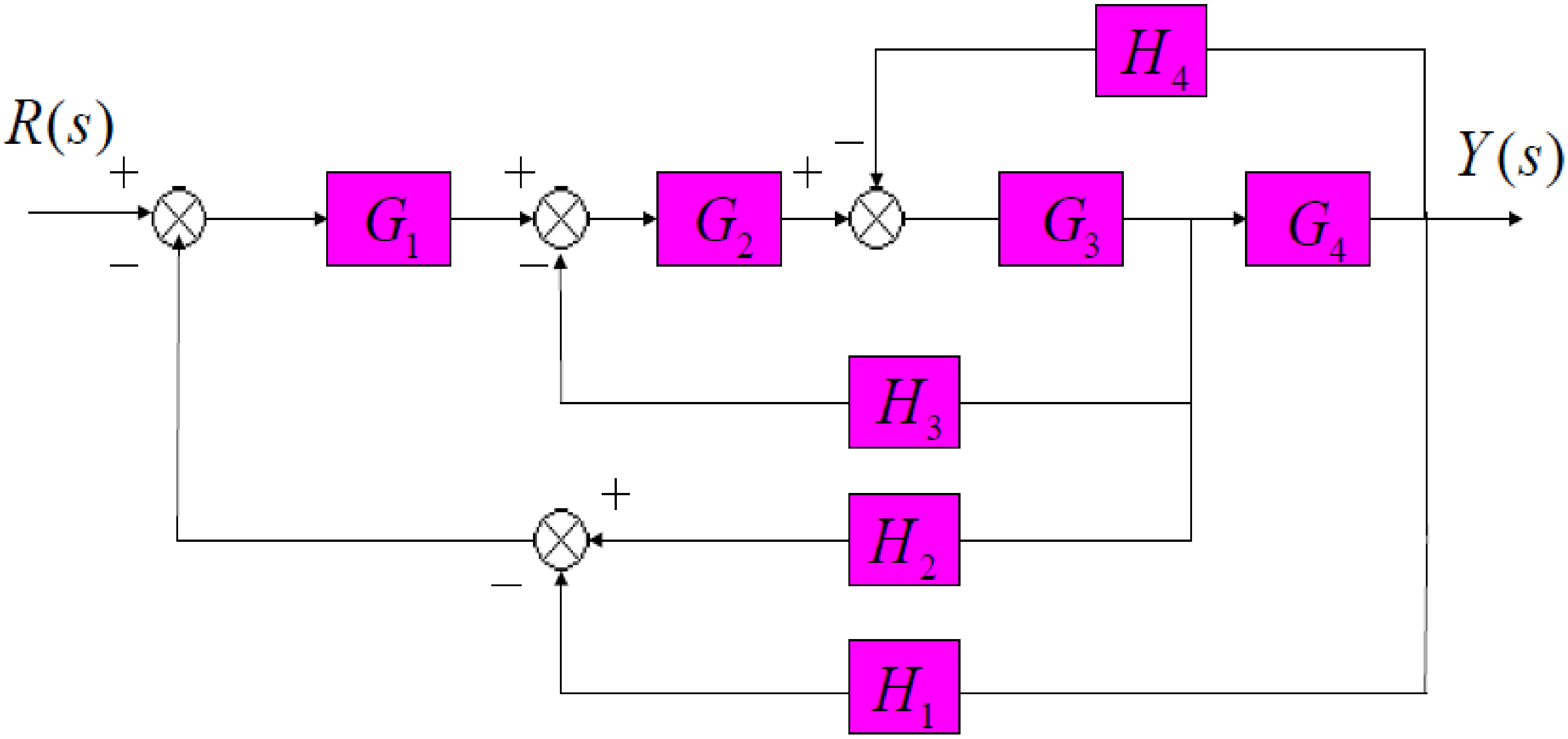


Using rule 6

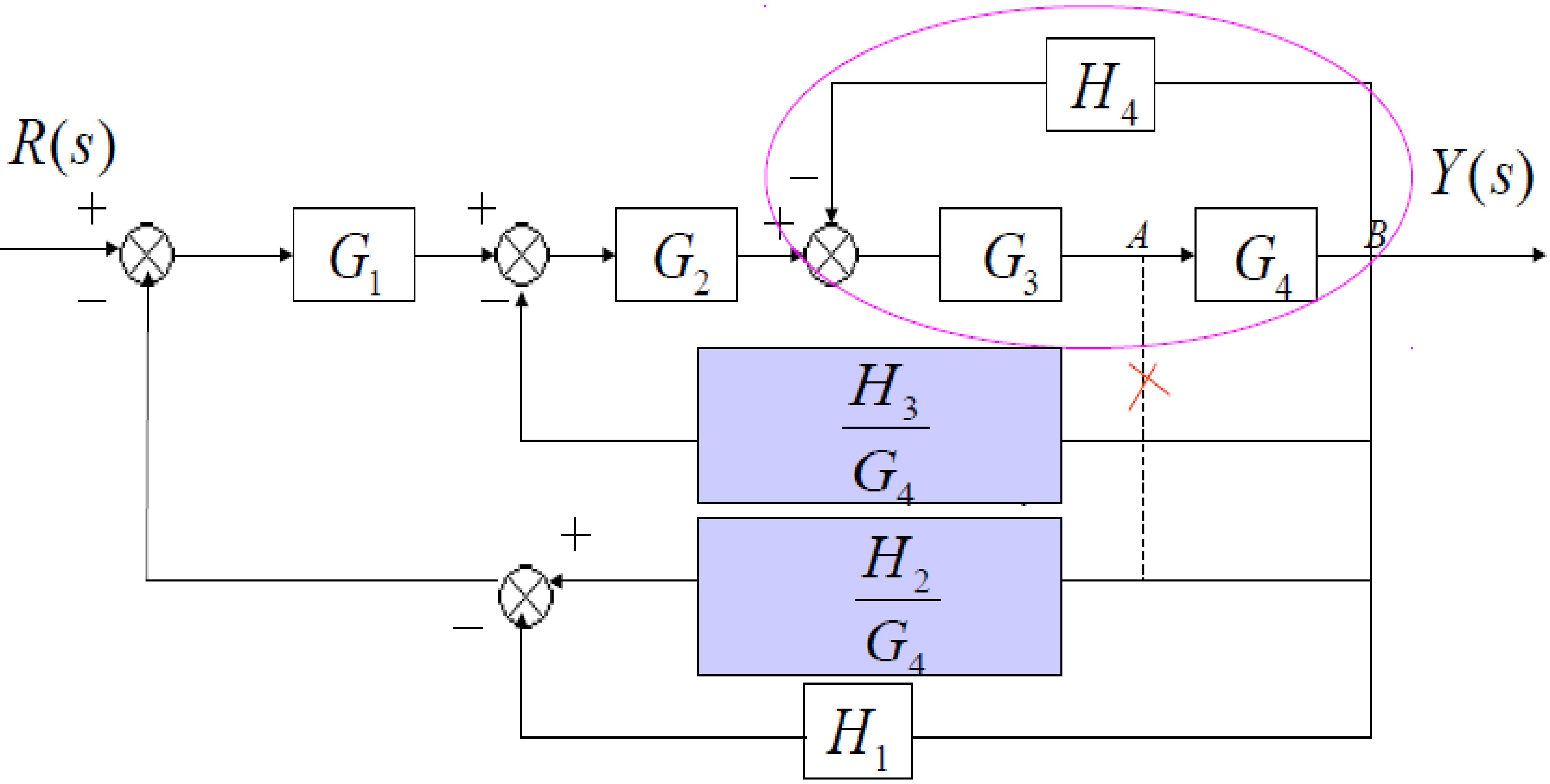


$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1G_2}{1 + G_2H_2 + G_1G_2H_3 + G_1H_1 + G_1G_2H_1H_2}$$

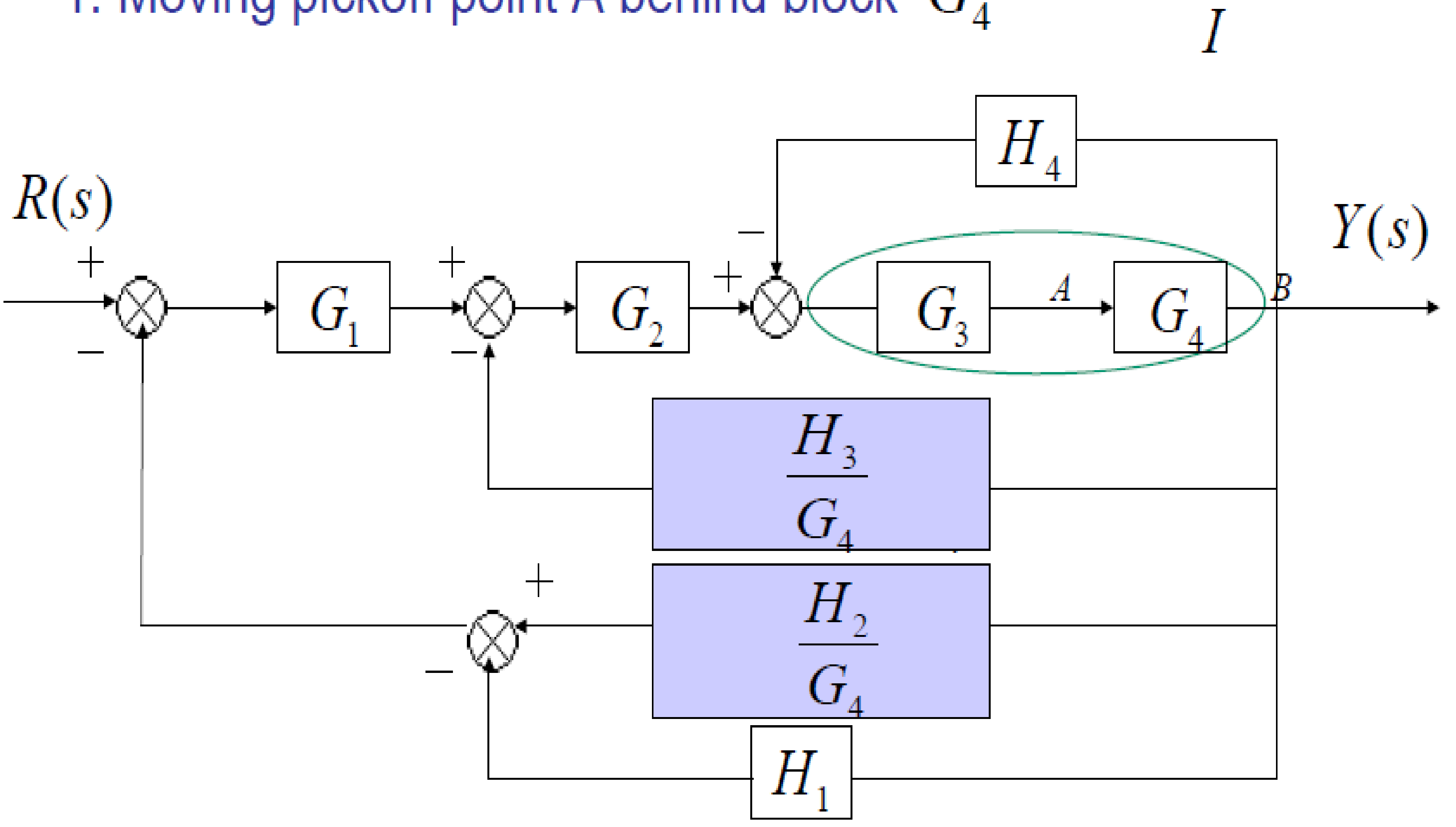
Ex.4: Find the transfer function of the following system using block diagram reduction techniques



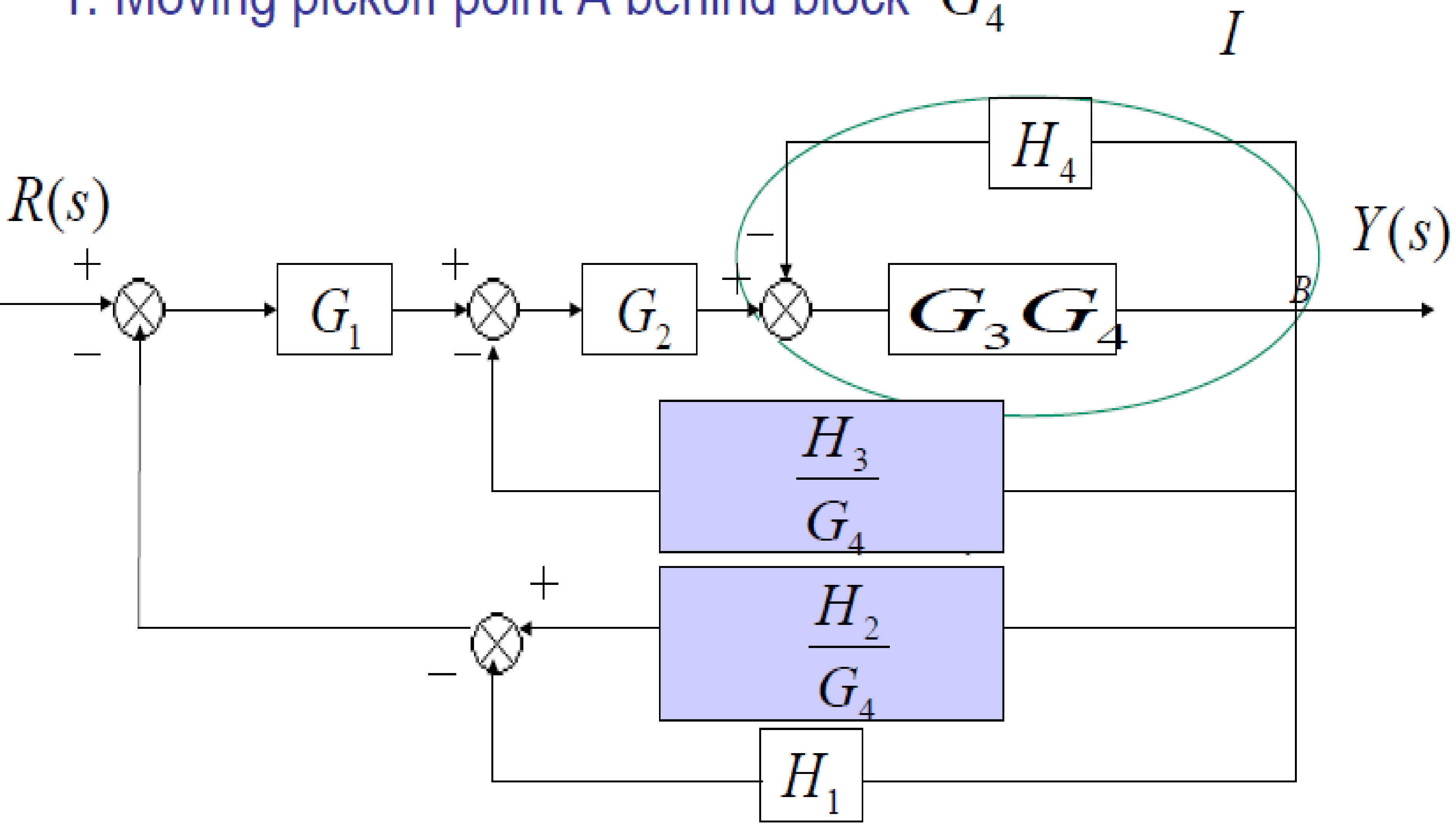
1. Moving pickoff point A behind block G_4



1. Moving pickoff point A behind block G_4

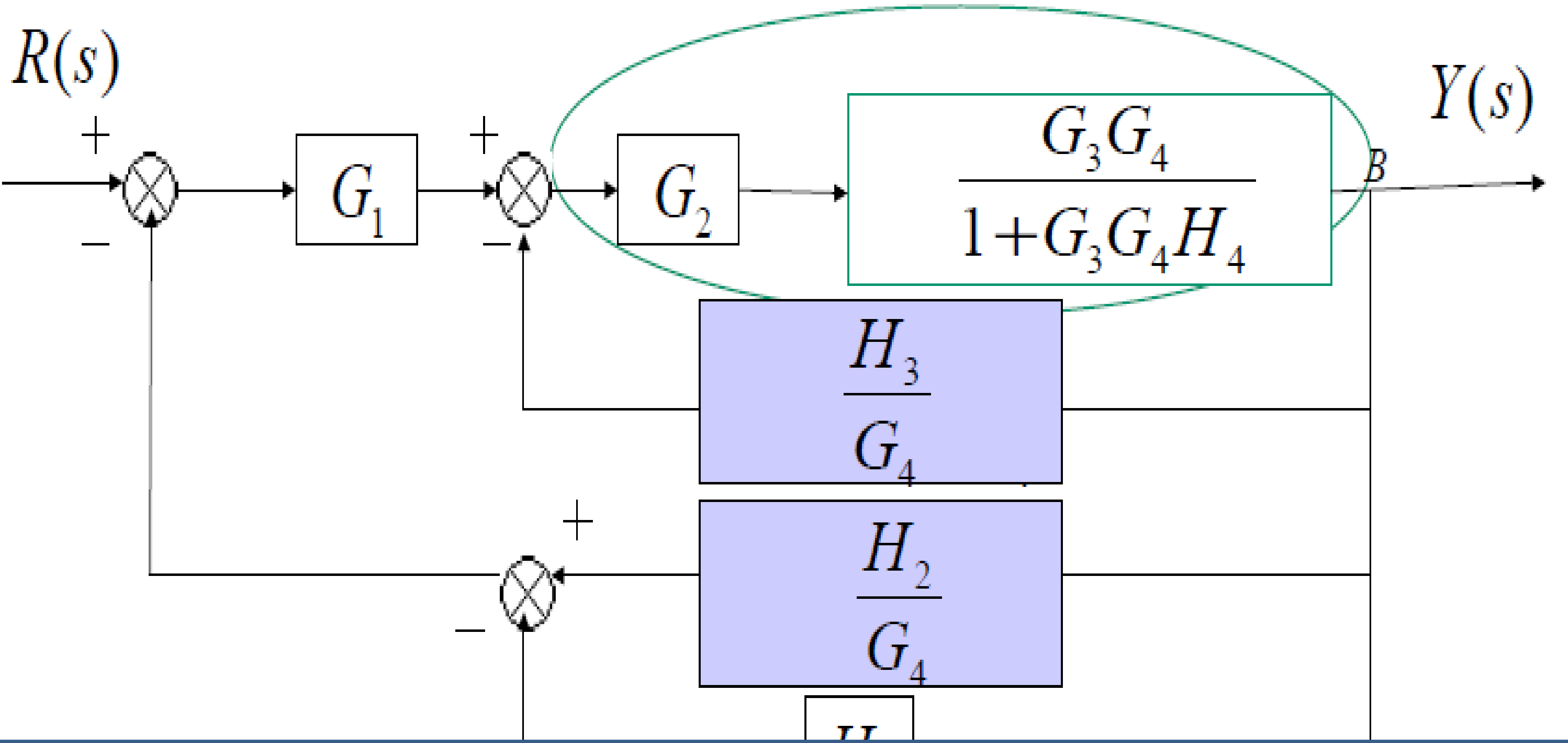


1. Moving pickoff point A behind block G_4

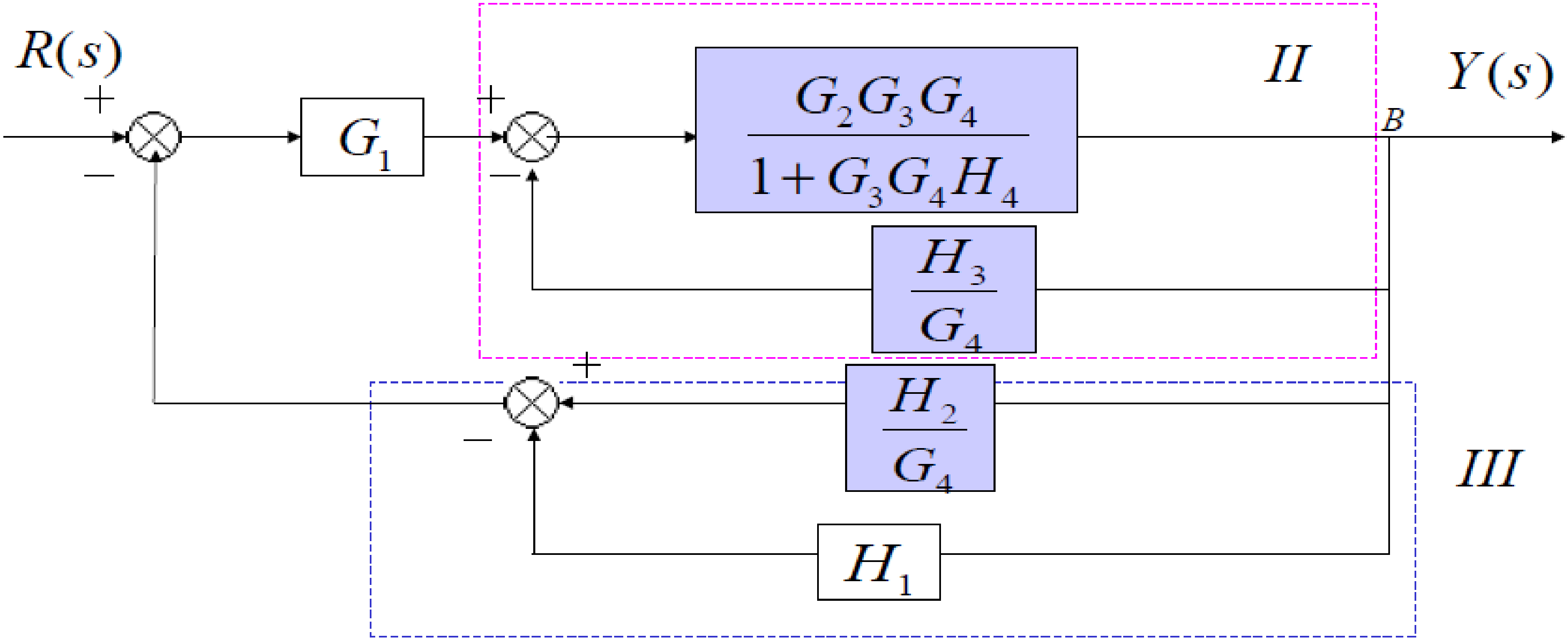


1. Moving pickoff point A behind block G_4

I



2. Eliminate loop I and Simplify



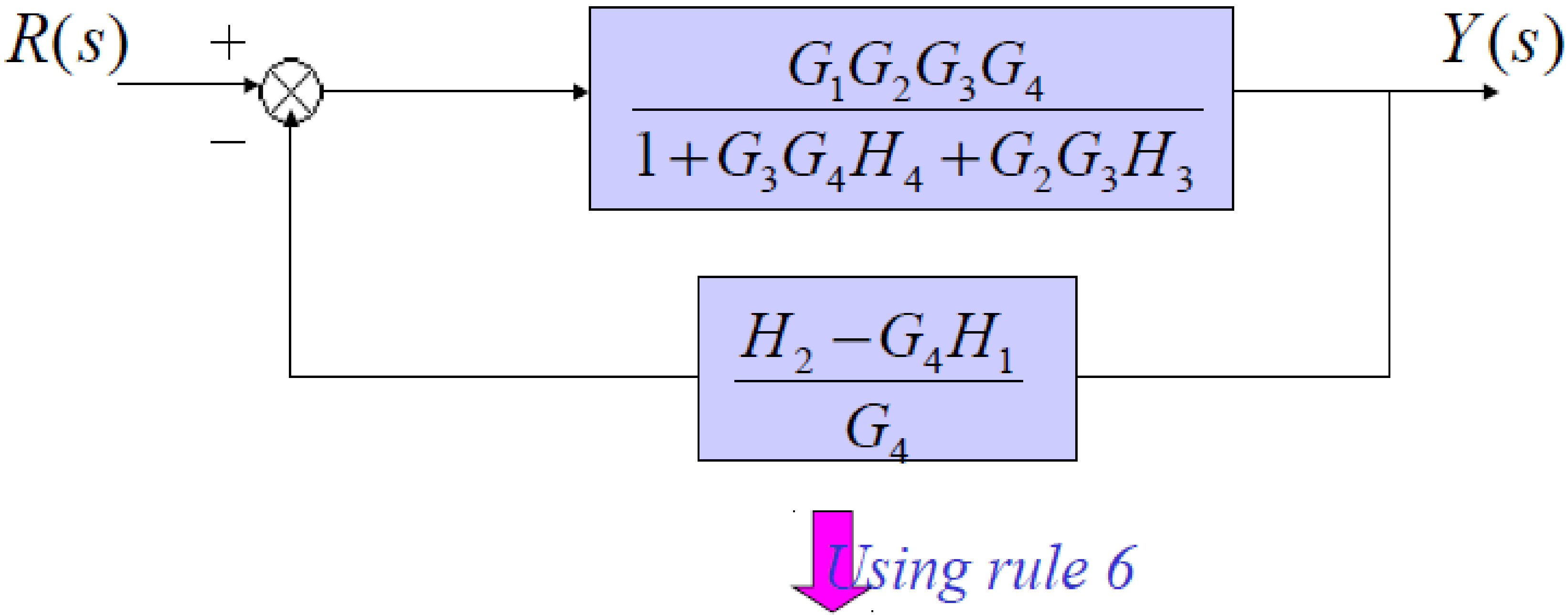
II feedback

$$\frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}$$

III Not feedback

$$\frac{H_2-G_4H_1}{G_4}$$

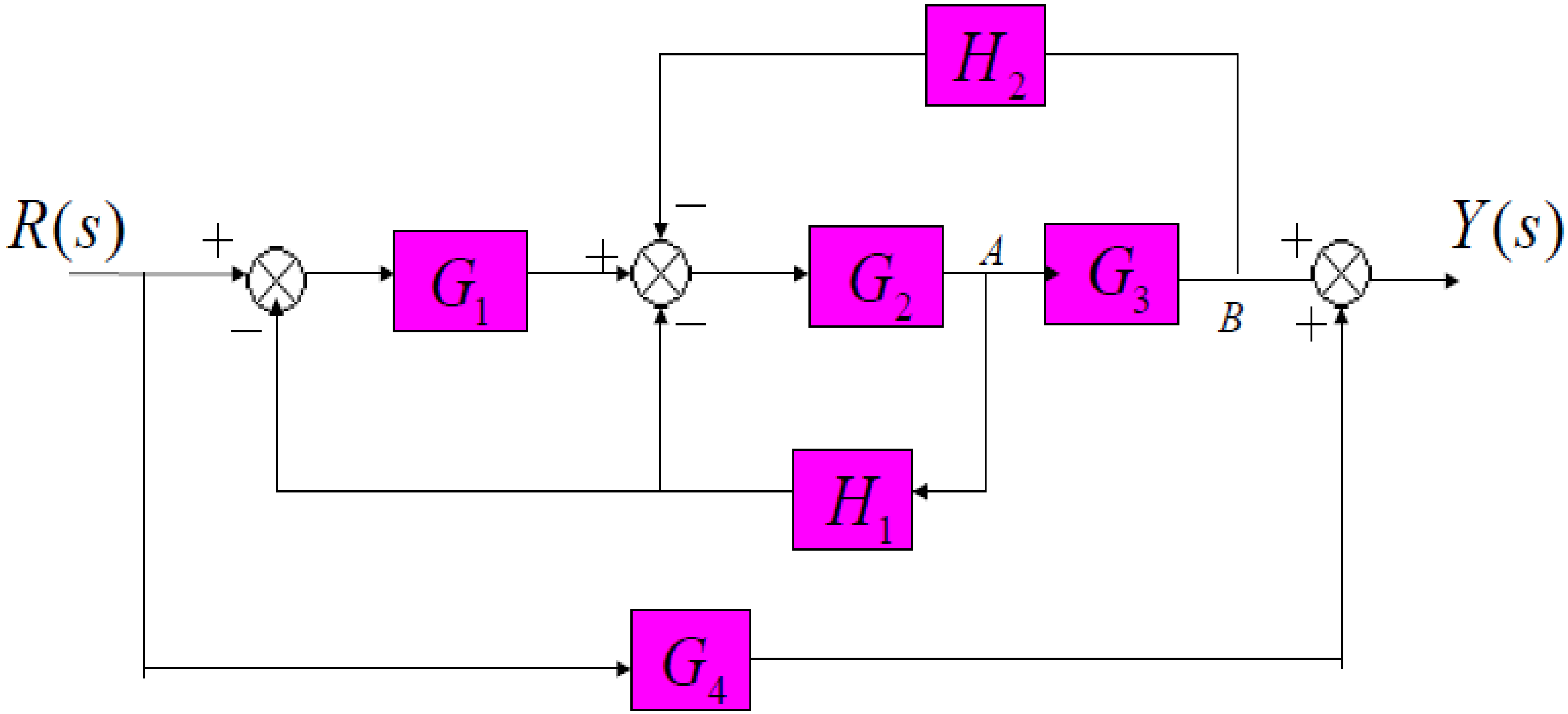
3. Eliminate loop II & III



$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

Skill Assessment Exercise 1

Find the transfer function of the following system using block diagram reduction techniques



Signal Flow Graphs

Outline

- Introduction to Signal Flow Graphs
  - Definitions
  - Terminologies
  - Examples
- Mason's Gain Formula
  - Examples
- Signal Flow Graph from Block Diagrams
- Design Examples

Introduction

- Alternative method to block diagram representation, developed by Samuel Jefferson Mason.

- Advantage: **Mason's gain formula** is used to find the overall gain/Transfer function of the system.
- Simpler than tedious Block diagram reduction method

- A signal-flow graph consists of a network in which nodes are connected by directed branches.

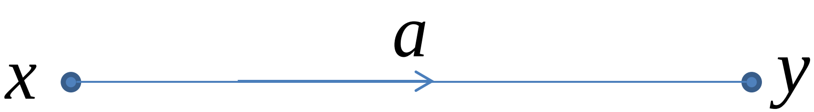
- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

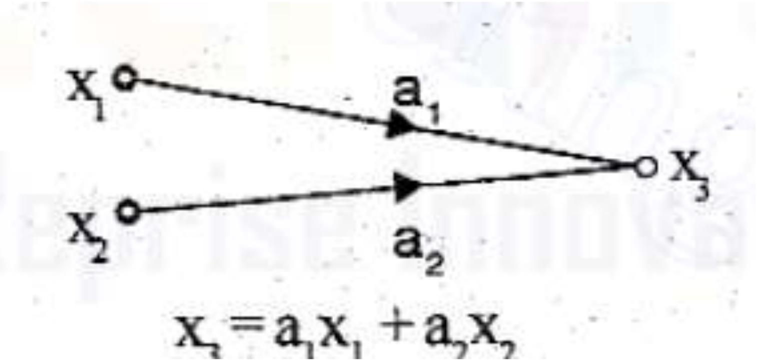
Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

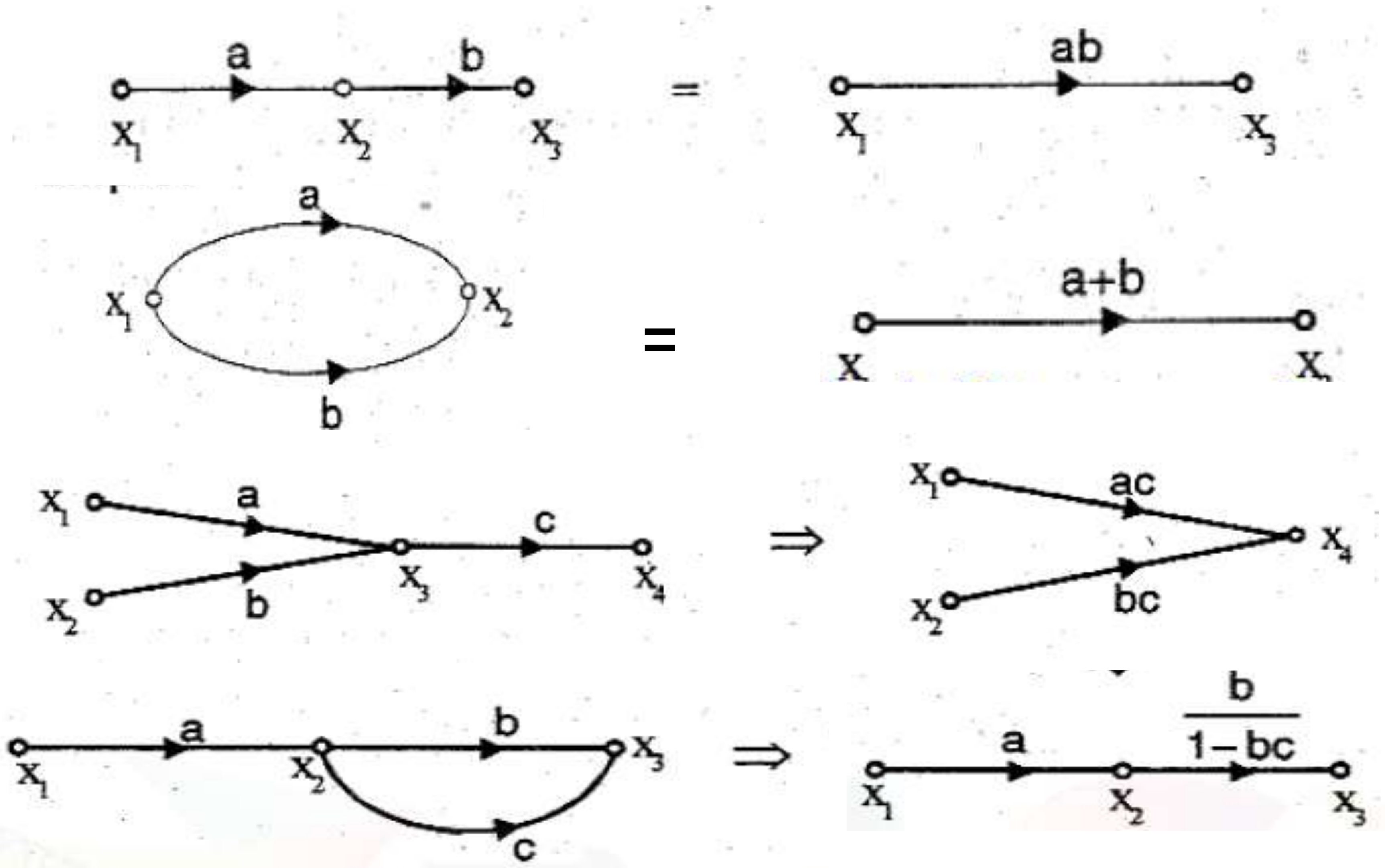
- The signal flow graph of the equati





- Every variable in a signal flow graph is represented by a **Node**.
- Every transmission function in a signal flow graph is represented by a **Branch**.

Signal-Flow Graph Models

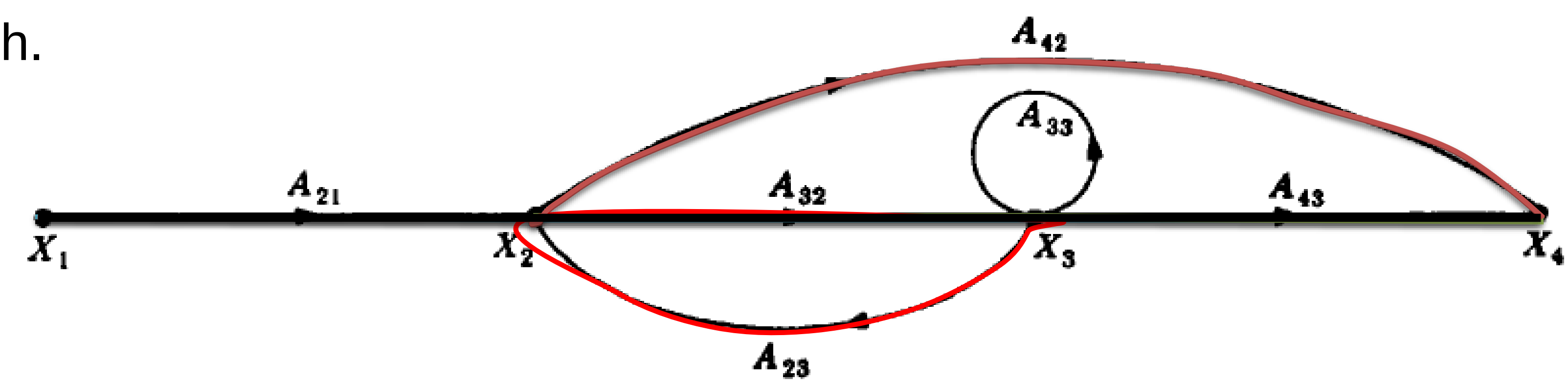


Terminologies

- An **input node** or source contain only the outgoing branches. i.e., X_1
- An **output node** or sink contain only the incoming branches. i.e., X_4
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e.,

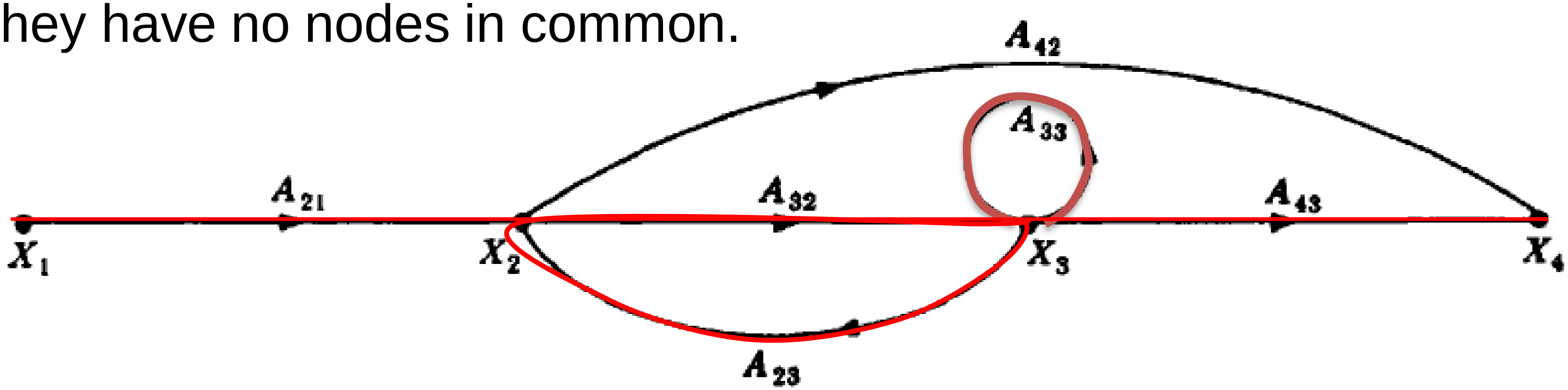
X_1 *to* X_2 *to* X_3 *to* X_4 X_1 *to* X_2 *to* X_4 X_2 *to* X_3 *to* X_4

- A **forward path** is a path from the input node to the output node. i.e., X_1 *to* X_2 *to* X_3 *to* X_4 , and X_1 *to* X_2 *to* X_4 , are forward paths.
- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; X_2 *to* X_3 **and back to** X_2 is a feedback path.

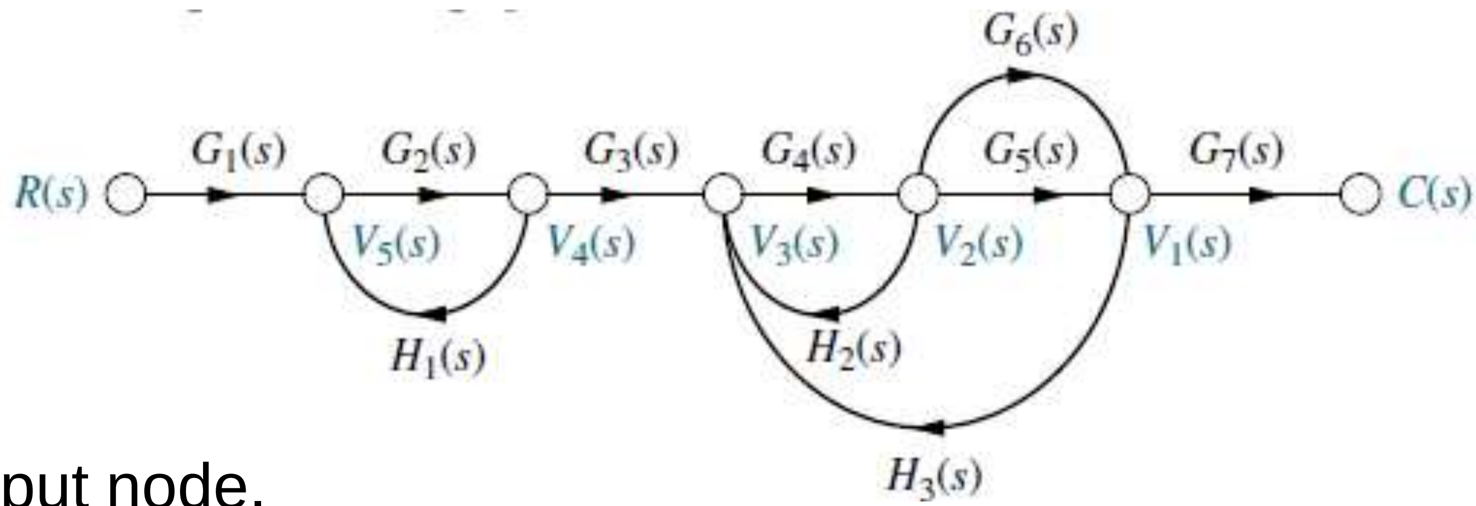


Terminologies

- A **self-loop** is a feedback loop consisting of a single branch. i.e.; A_{33} is a self loop.
- The **gain** of a branch is the transmission function of that branch.
- The **path gain** is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X_1 *to* X_2 *to* X_3 *to* X_4 is $A_{21}A_{32}A_{43}$
- The **loop gain** is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.

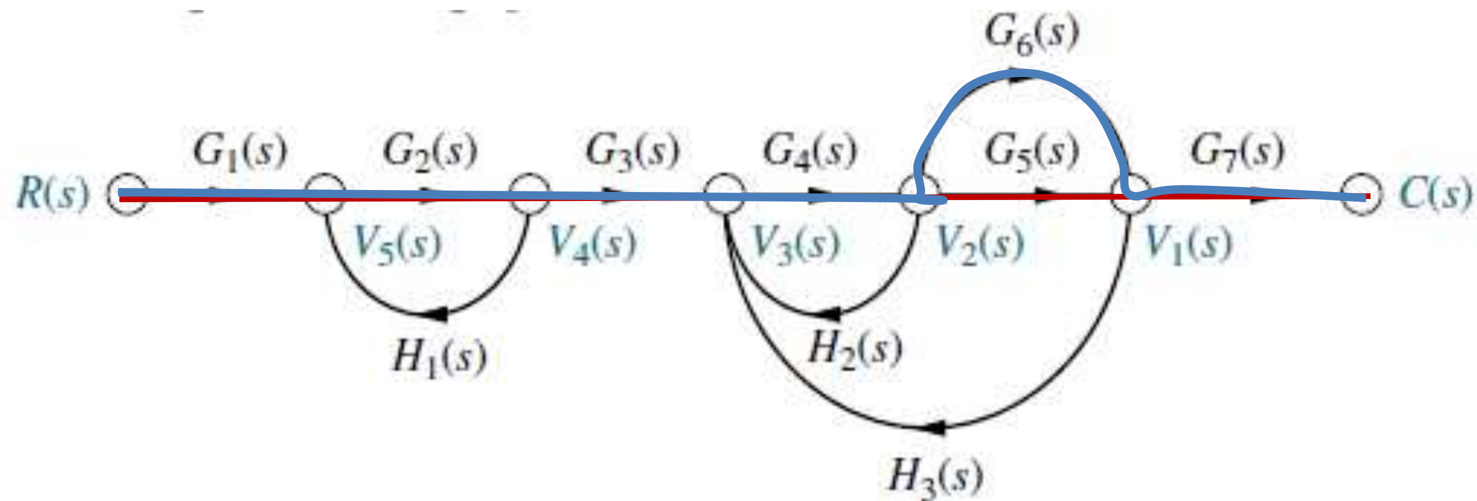


Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths (loops).
e) Determine the loop gains of the feedback loops.
f) Determine the path gains of the forward paths.

Consider the signal flow graph below and identify the following



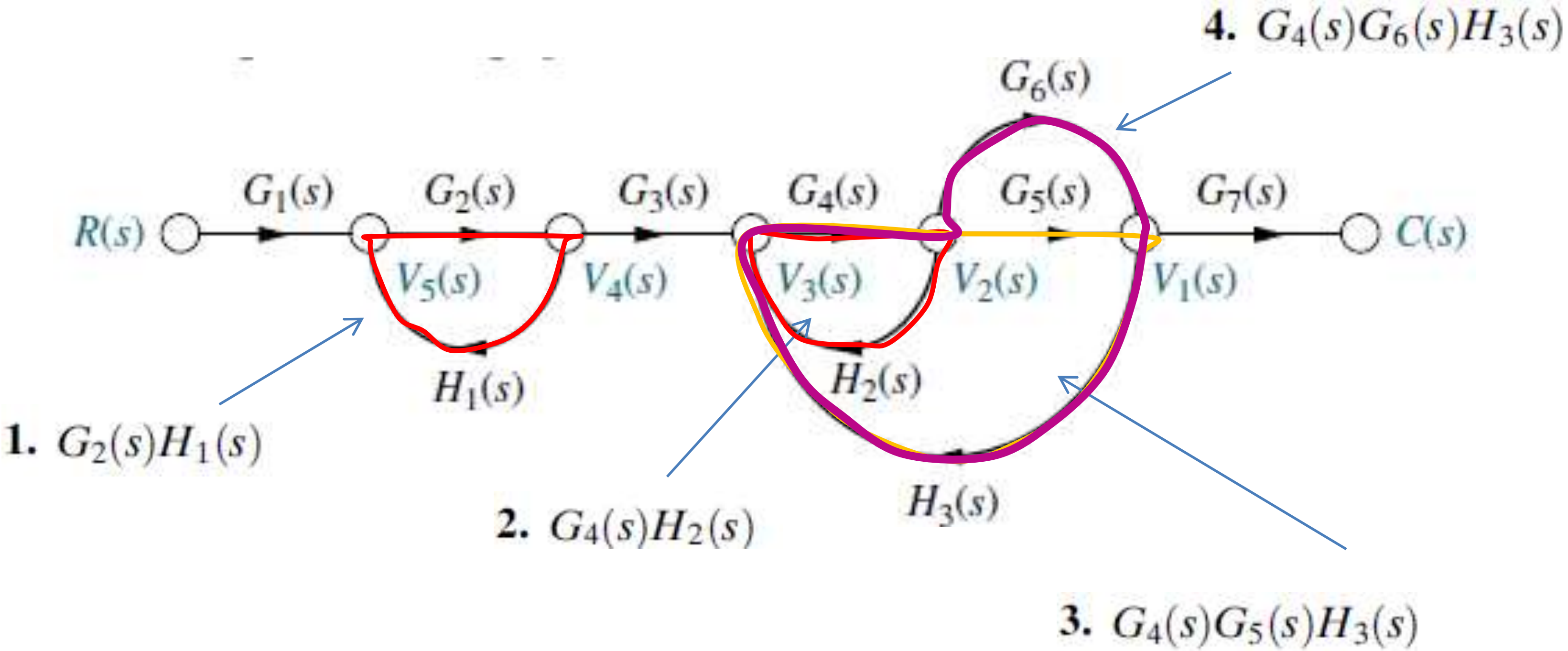
- There are two forward path gains;

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$

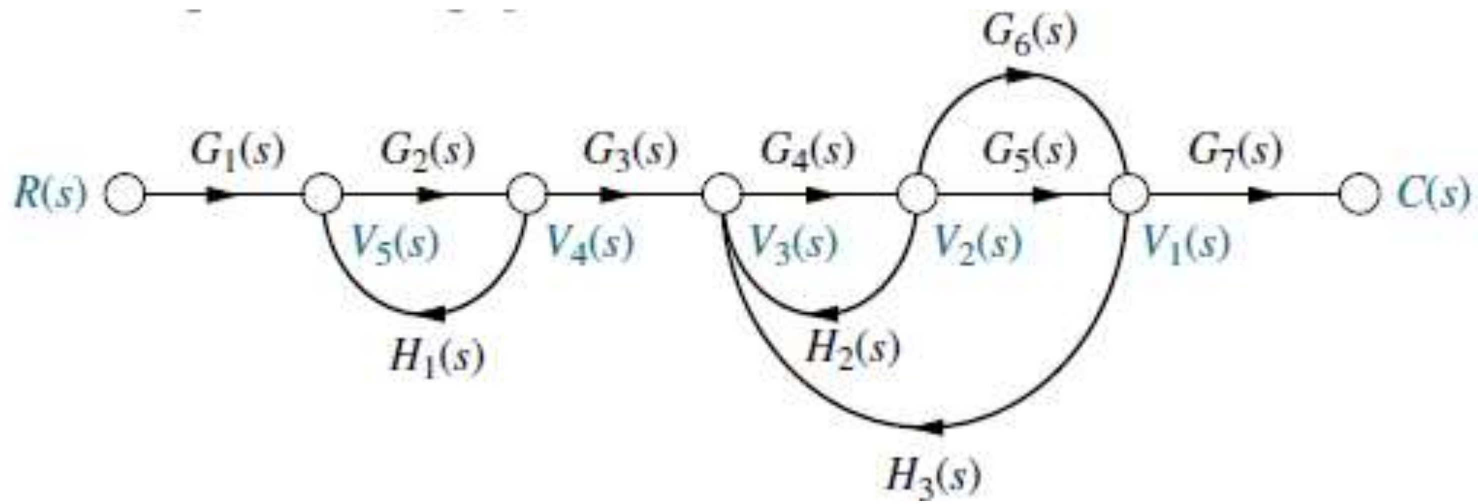
2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

Consider the signal flow graph below and identify the following

- There are four loops



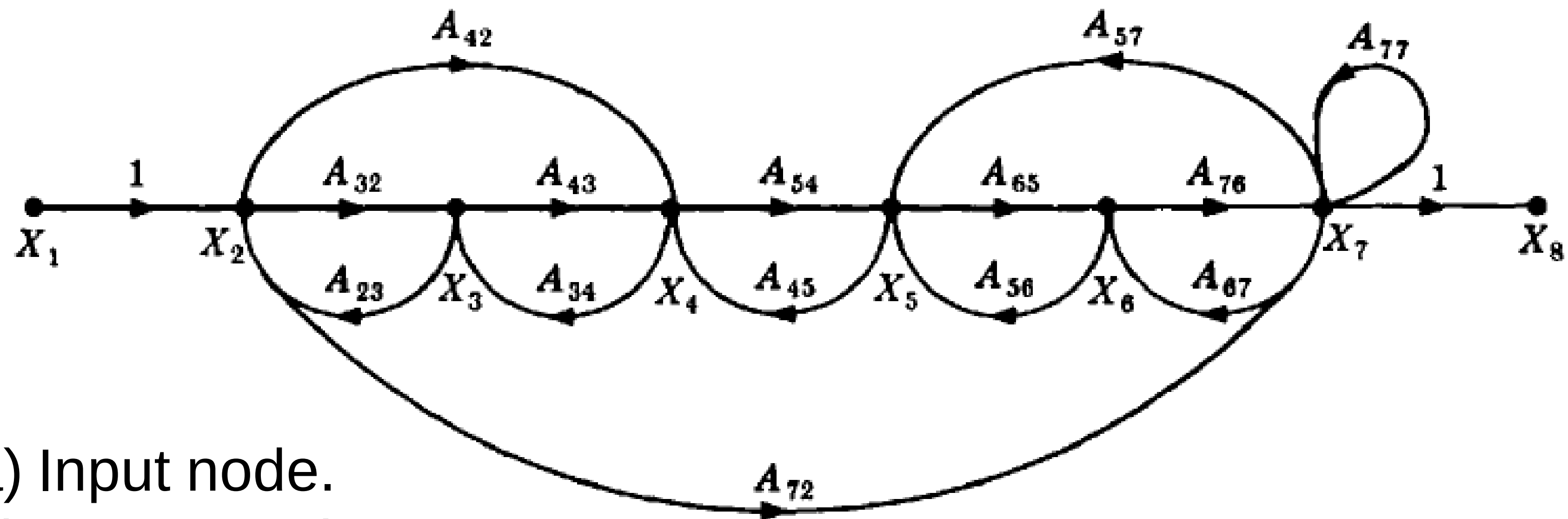
Consider the signal flow graph below and identify the following



- Nontouching loop gains;

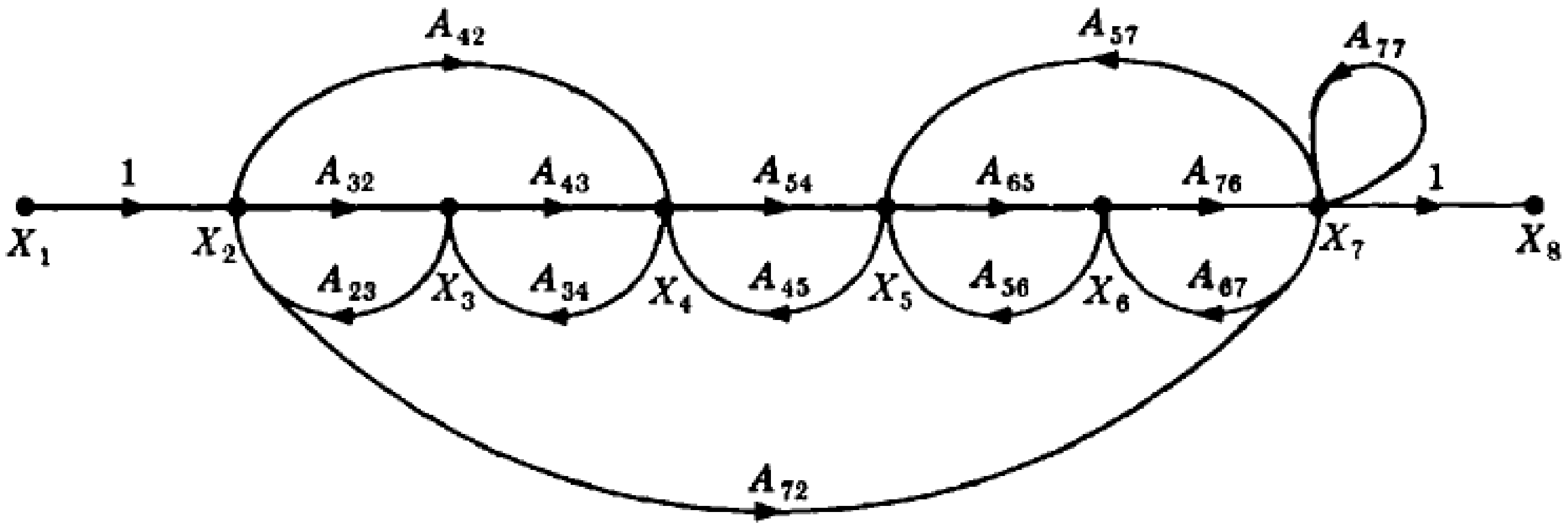
1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths.
e) Self loop.
f) Determine the loop gains of the feedback loops.
g) Determine the path gains of the forward paths.

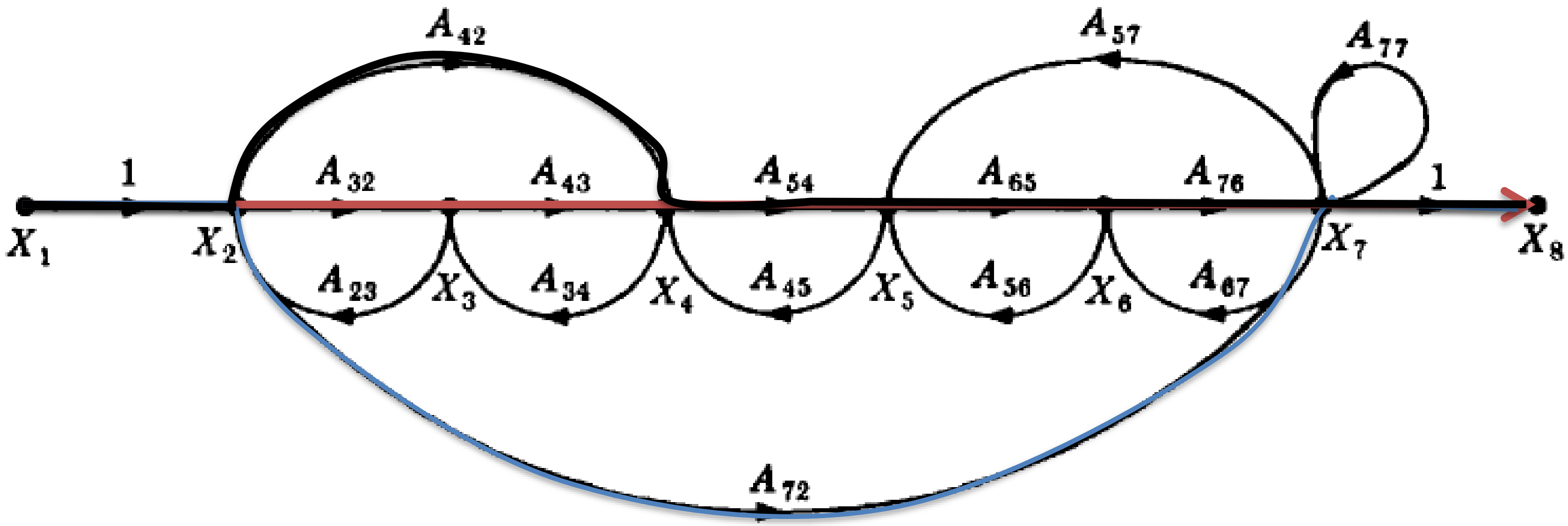
Input and output Nodes



a) Input no X_1

b) Output node X_8

(c) Forward Paths

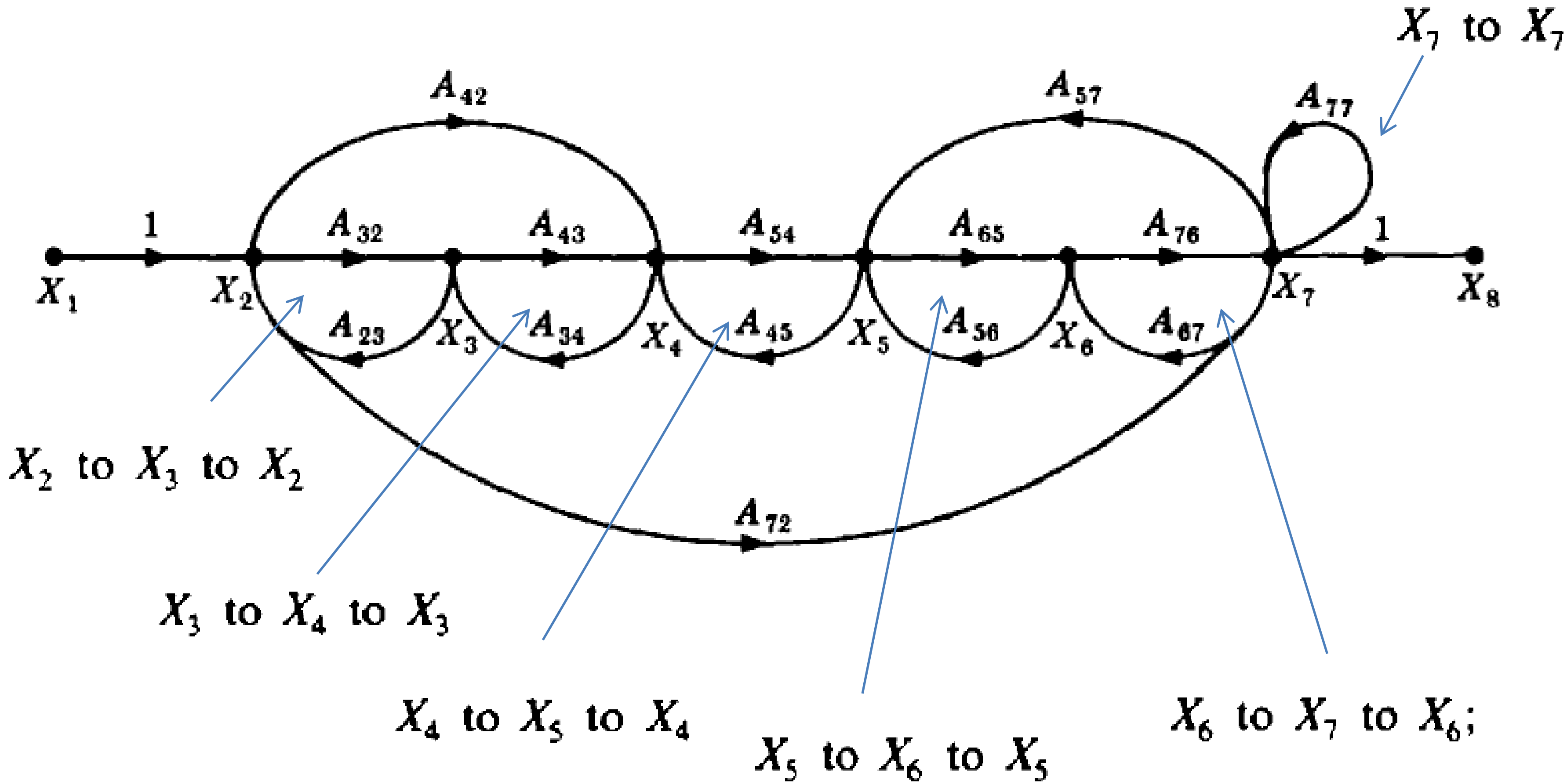


X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8

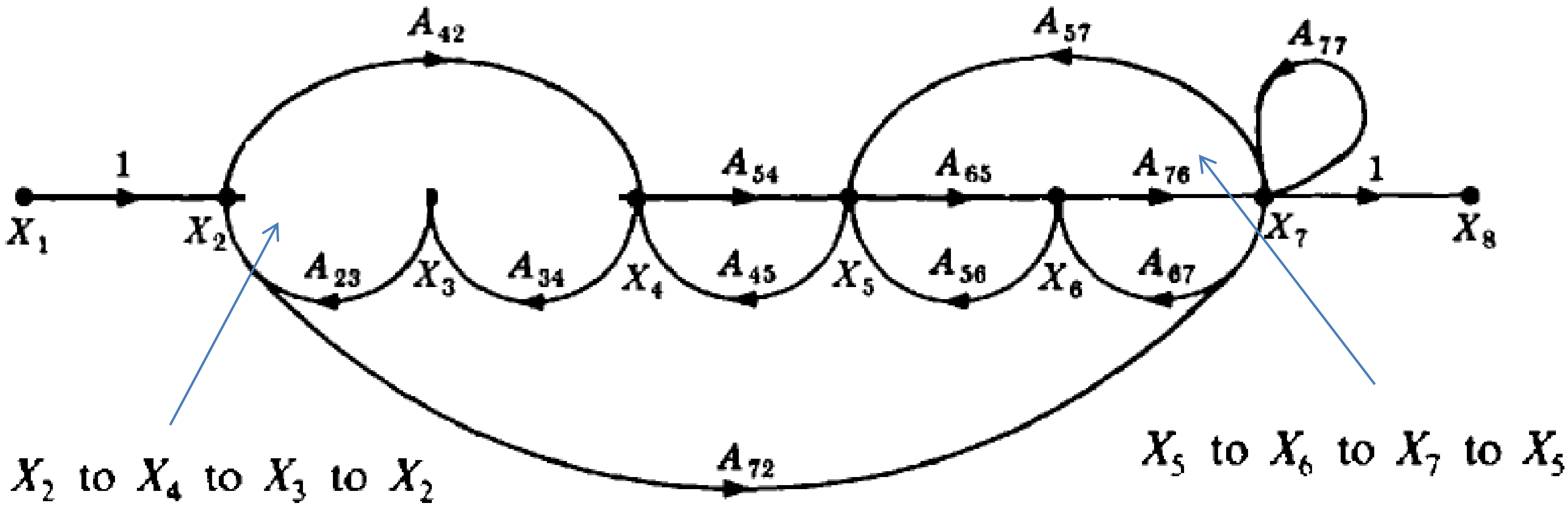
X_1 to X_2 to X_7 to X_8

X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

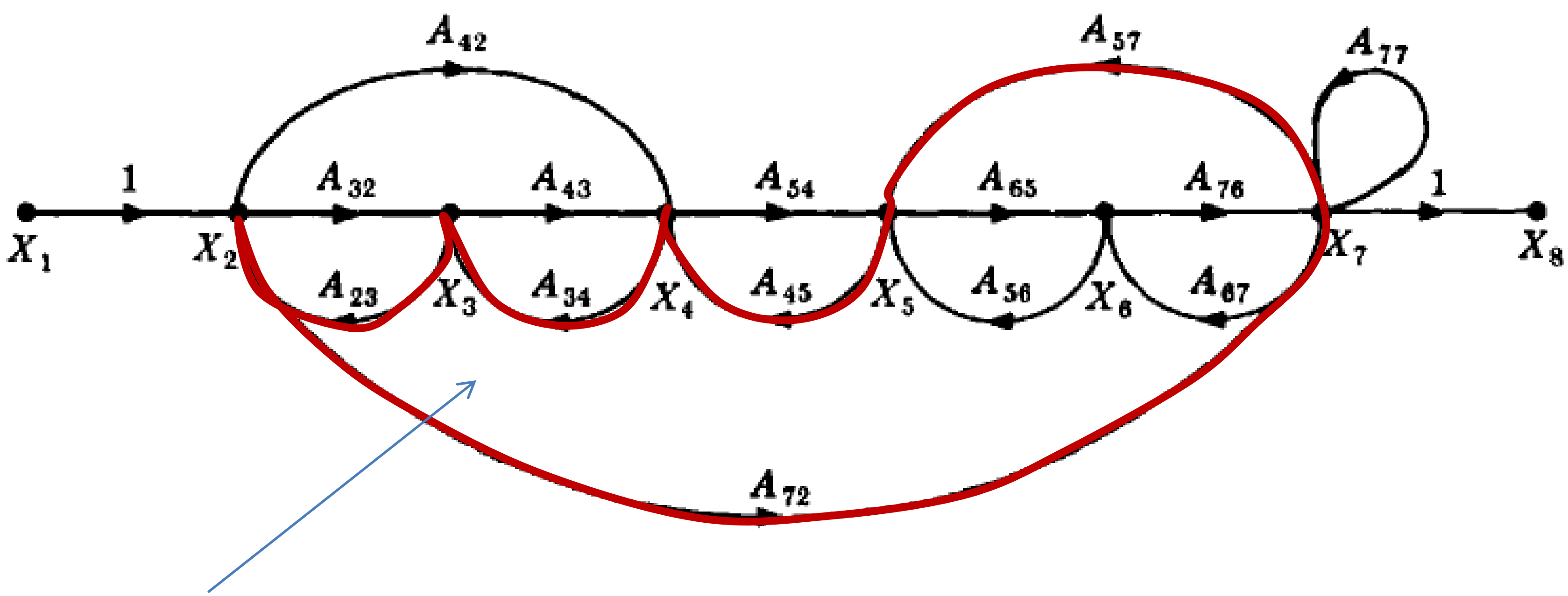
(d) Feedback Paths or Loops



(d) Feedback Paths or Loops

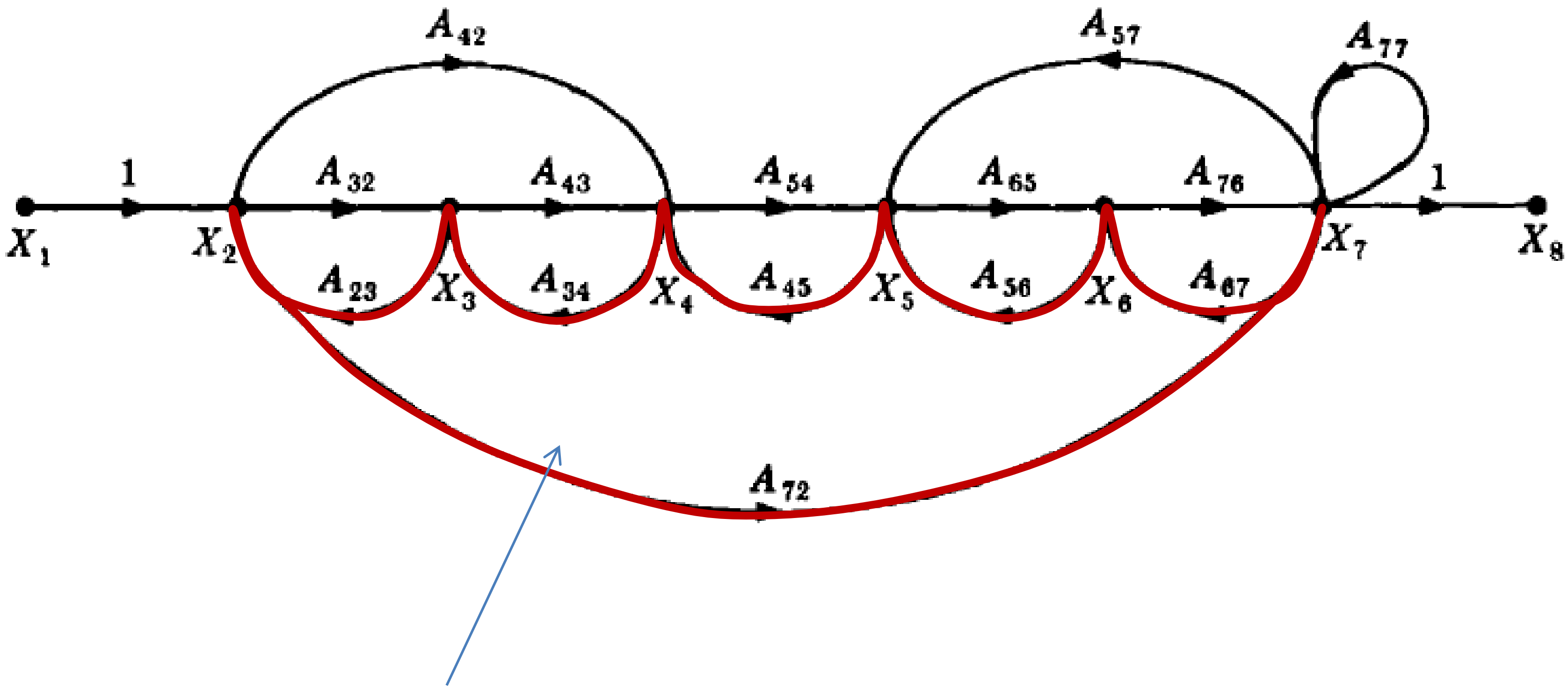


(d) Feedback Paths or Loops



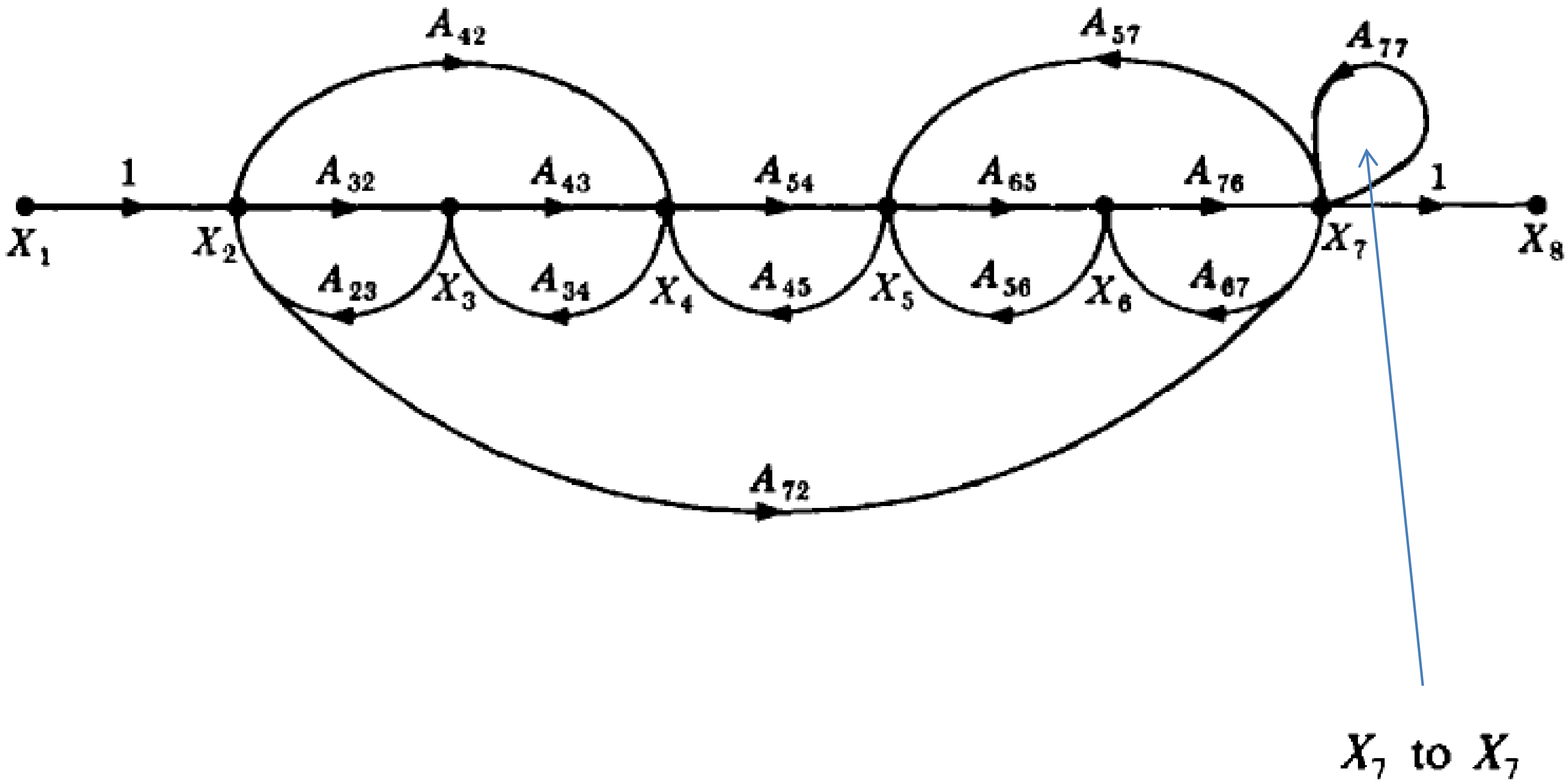
X_2 to X_7 to X_5 to X_4 to X_3 to X_2

(d) Feedback Paths or Loops



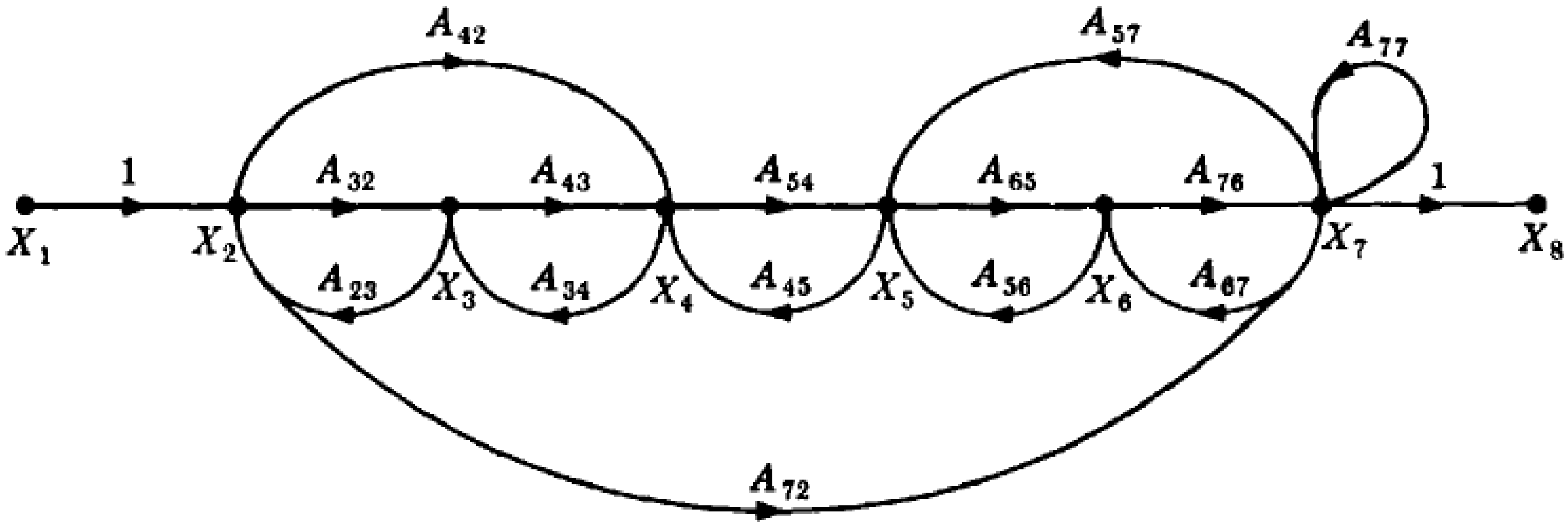
X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X_2

(e) Self Loop(s)



A_{42}
A_{57}
A_{77}
1
A_{32}
A_{43}
A_{54}
A_{65}
A_{76}
1
X_1
X_2
X_7
X_8
A_{23}
X_3
A_{34}
X_4
A_{45}
X_5
A_{56}
X_6
A_{67}
A_{72}
X_7 to X_7

(f) Loop Gains of the Feedback Loops



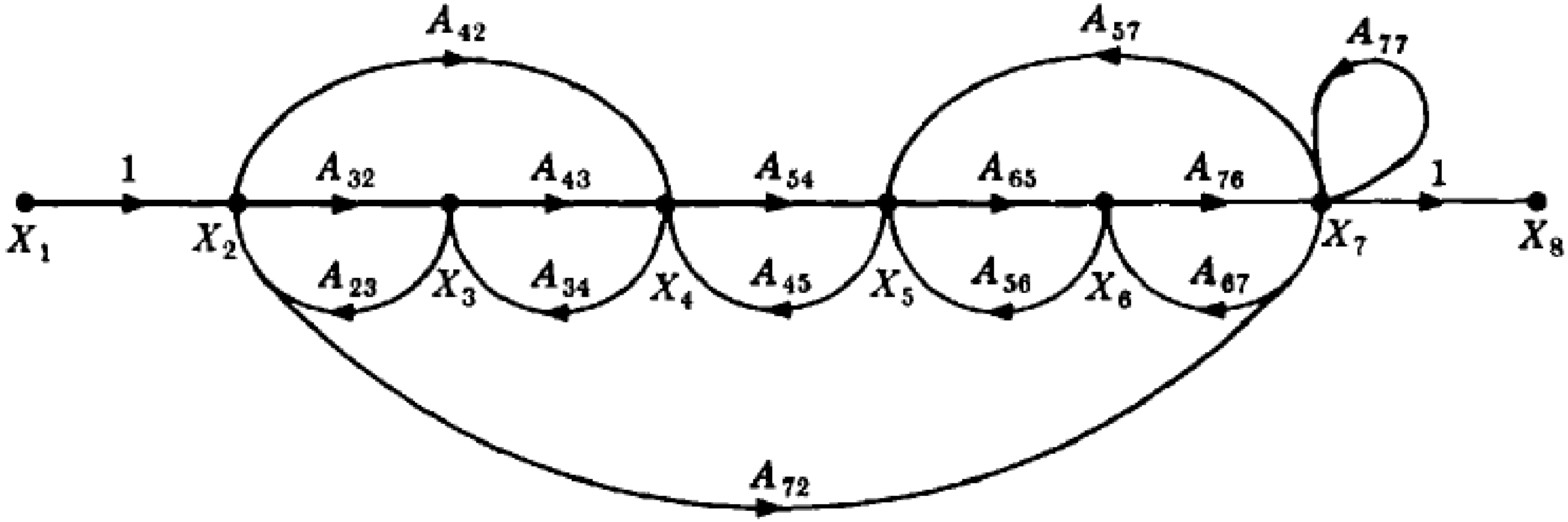
$A_{32}A_{23}$ $\quad$ $A_{76}A_{67}$; $\quad$ $A_{72}A_{57}A_{45}A_{34}A_{23}$;

$A_{43}A_{34}$; $\quad$ $A_{65}A_{76}A_{57}$; $\quad$ $A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$

$A_{54}A_{45}$; $\quad$ A_{77}

$A_{65}A_{56}$ $\quad$ $A_{42}A_{34}A_{23}$

(g) Path Gains of the Forward Paths



$A_{32} A_{43} A_{54} A_{65} A_{76}$

A_{72}

$A_{42} A_{54} A_{65} A_{76}$

Mason's Gain Formula:

- The transfer function, ***C(s)/R(s),*** of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{i=1}^{n} P_i \Delta_i$$

Where

n = number of forward paths.

P_i = the i^{th} forward-path gain.

Δ = Determinant of the system

Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function.

Mason's Rule:

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{i=1}^{n} P_i \Delta_i$$

Pi = Forward path gains

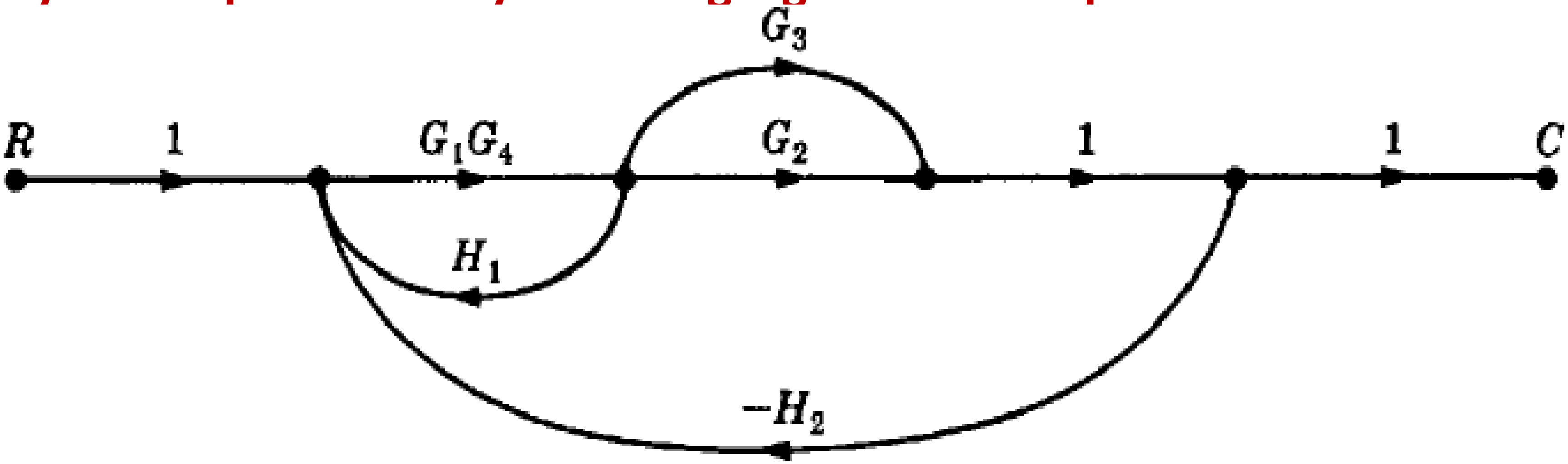
Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

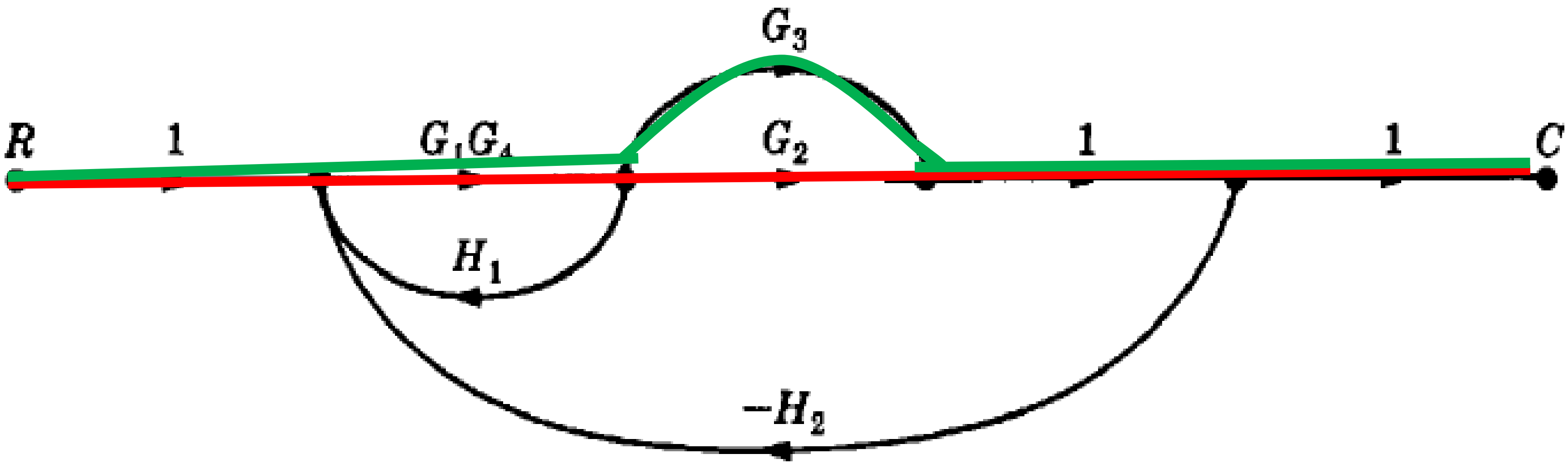
Δ_i = value of Δ for the part of the block diagram that does not touch the i-th forward path (Δ_i = 1 if there are no non-touching loops to the i-th path.)

Systematic approach

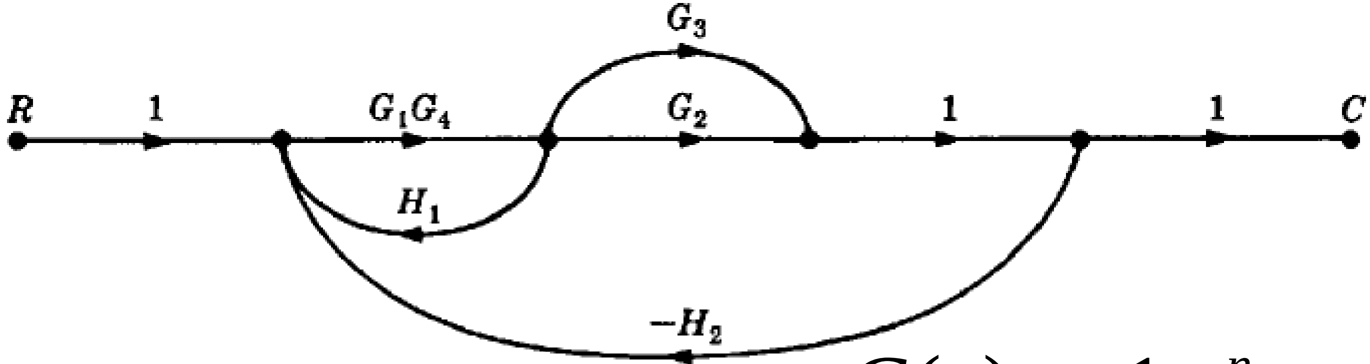
1. Calculate forward path gain P_i for each forward path *i.*
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time etc.
5. Calculate Δ from steps 2,3,4 and 5
6. Calculate Δ_i as portion of Δ not touching forward path *i*

Example #1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph





Example #1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



There are two forward paths:

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_{i=1}^{n} P_i\Delta_i$$

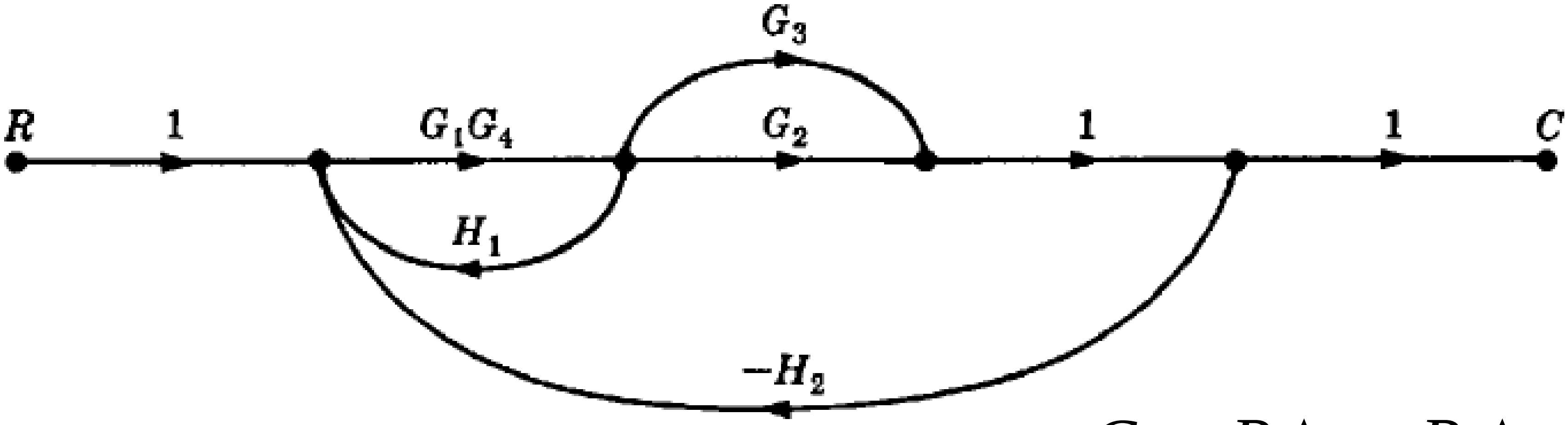
$P_1 = G_1G_2G_4$ $\qquad$ $P_2 = G_1G_3G_4$

Therefore,

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$



$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

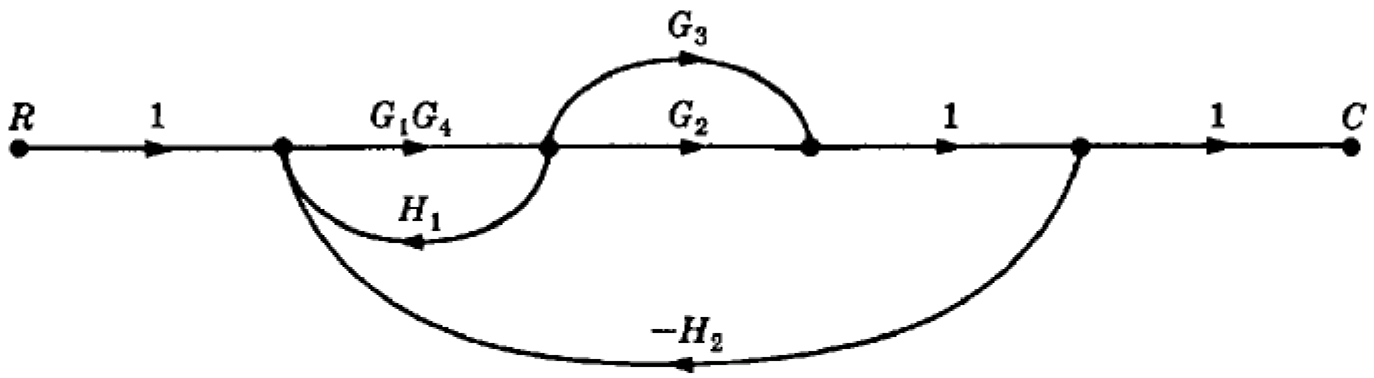
There are no non-touching loops, therefore

Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - (G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2)$$

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+...

$\Delta_1 = 1$

Eliminate forward path-2

Δ_2 = 1- (sum of all individual loop gains)+...

$\Delta_2 = 1$

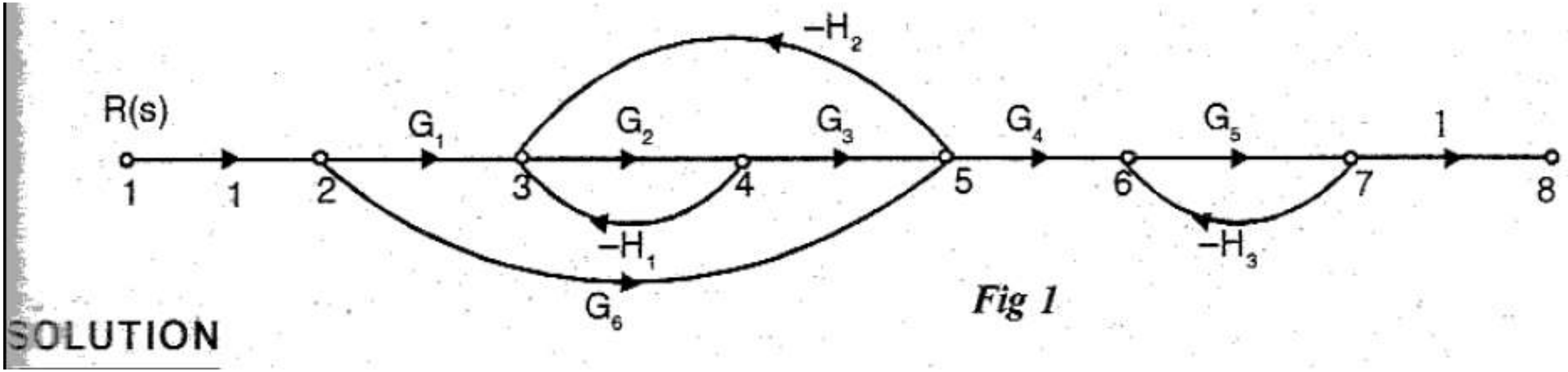
Example#1: Continue

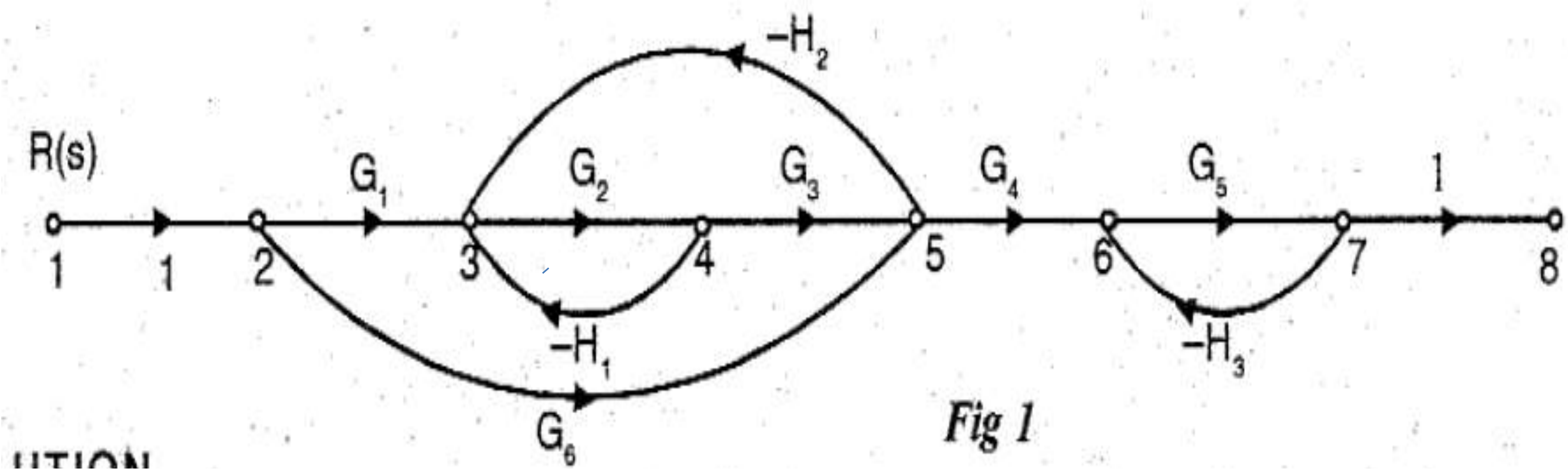
$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Transfer function:

$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Example #1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph.





There are two forward paths. $\therefore K = 2$

Let forward path gains be P_1 and P_2.

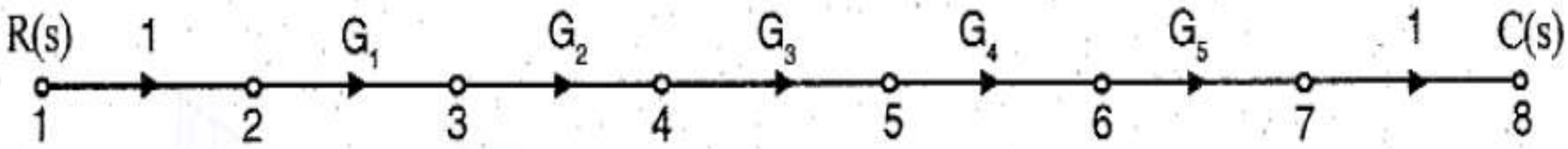


Fig 2 : Forward path-1.

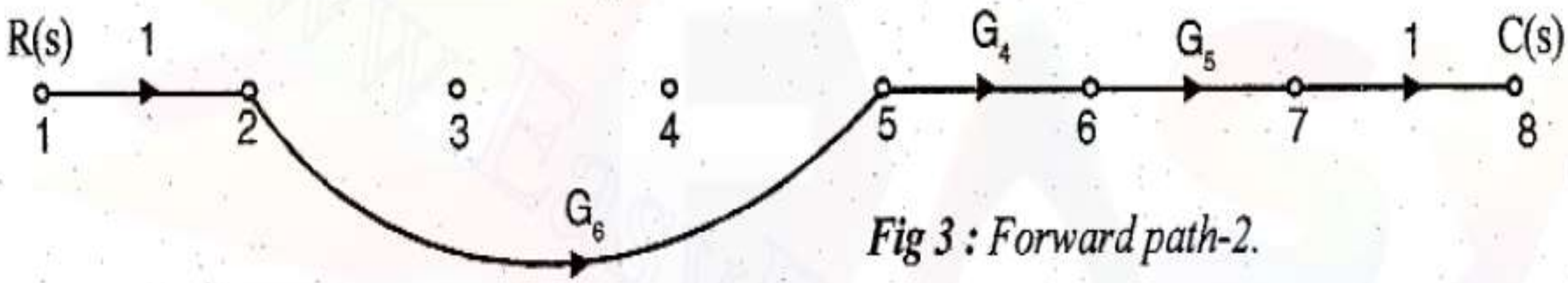


Fig 3 : Forward path-2.

Gain of forward path-1, $P_1 = G_1 G_2 G_3 G_4 G_5$

Gain of forward path-2, $P_2 = G_4 G_5 G_6$

Individual Loop Gain

There are three individual loops. Let individual loop gains be P_{11}, P_{21} and P_{31}.

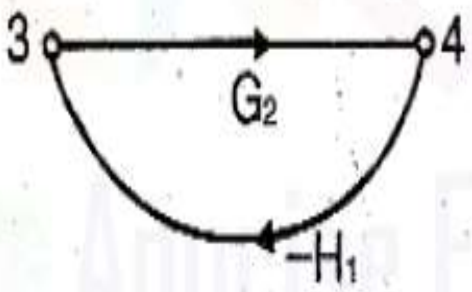


Fig 4 : Loop-1.

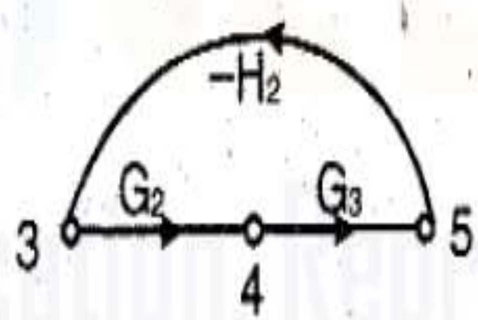


Fig 5 : Loop-2.

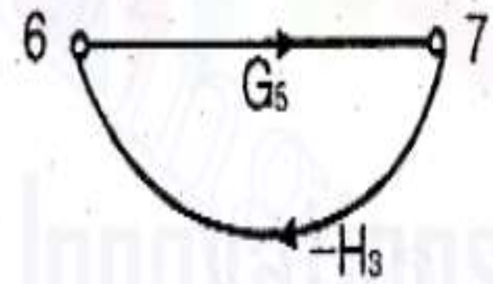


Fig 6 : Loop-3.

Loop gain of individual loop-1, $P_{11} = -G_2 H_1$

Loop gain of individual loop-2, $P_{21} = -G_2 G_3 H_2$

Loop gain of individual loop-3, $P_{31} = -G_5 H_3$

Gain Products of Two Non-touching Loops

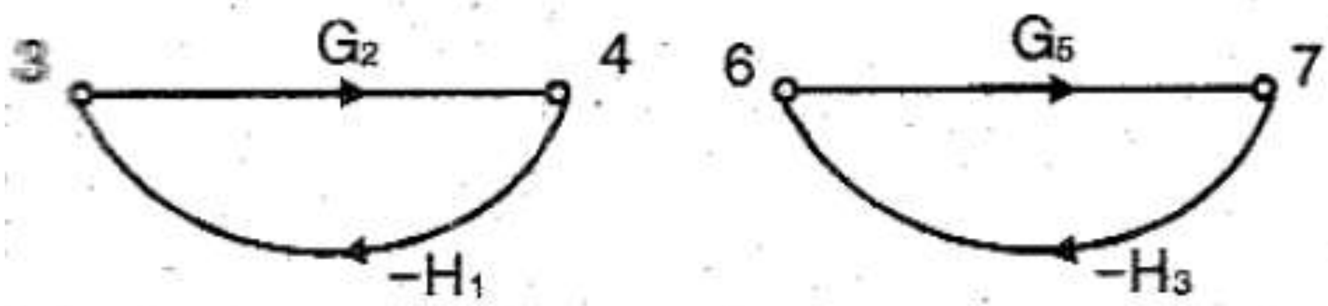


Fig 7 : First combination of 2 non-touching loops.

L12 = $(-G_2H_1)(-G_5H_3) = G_2G_5H_1H_3$

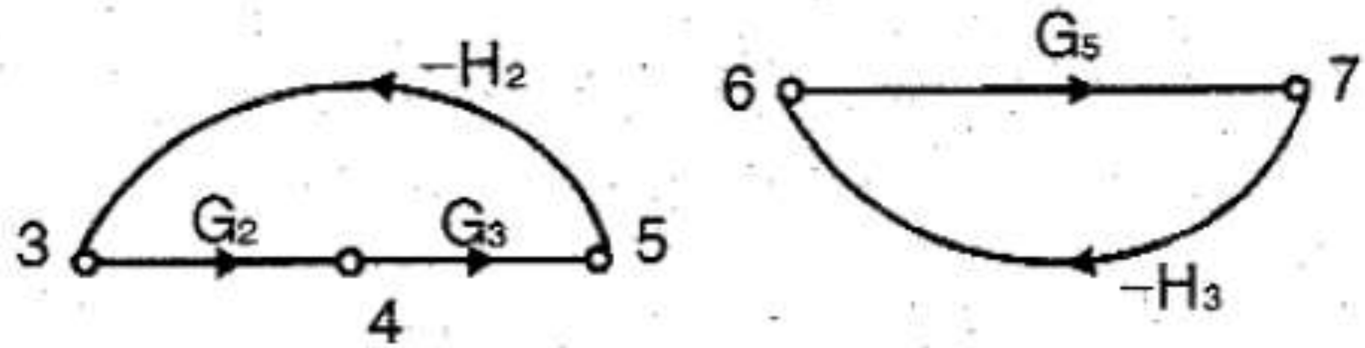


Fig 8 : Second combination of 2 non-touching loops.

L22 = $(-G_2G_3H_2)(-G_5H_3) = G_2G_3G_5H_2H_3$

Calculation of Δ and Δ_K

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

Since k=2

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_{12} + L_{22})$$

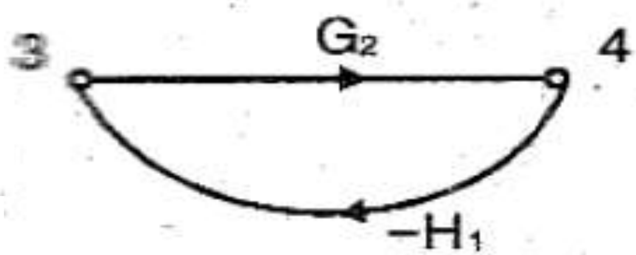
$$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward pat

The part of the graph which is non touching with second forward path is shown in fig 9.

$$\Delta_2 = 1 - (-G_2H_1) = 1 + G_2H_1$$



Transfer Function

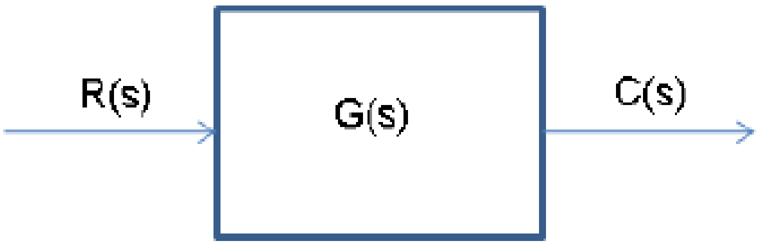
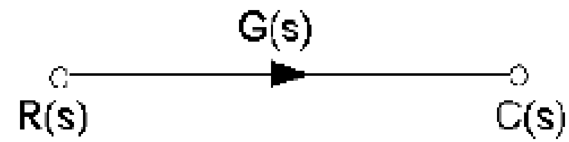
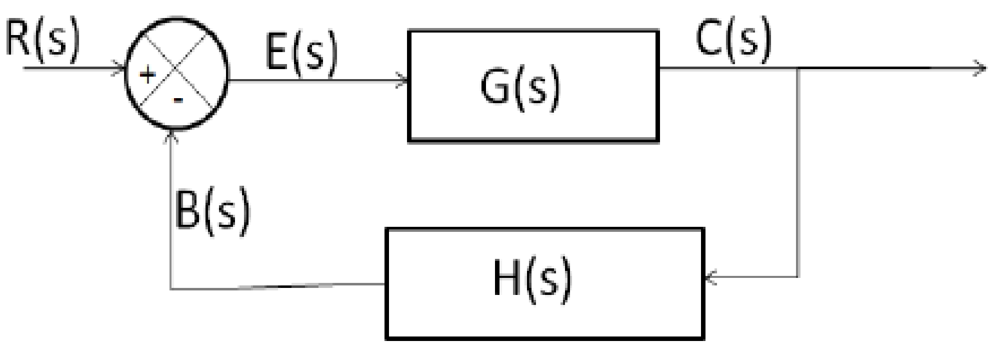
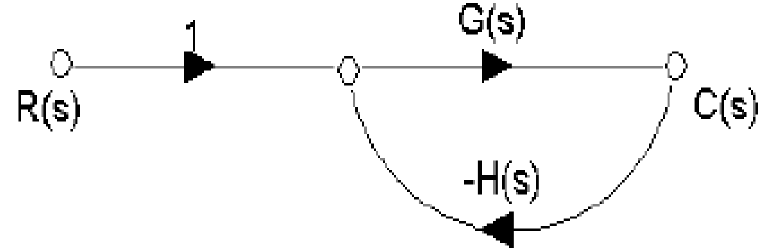
$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

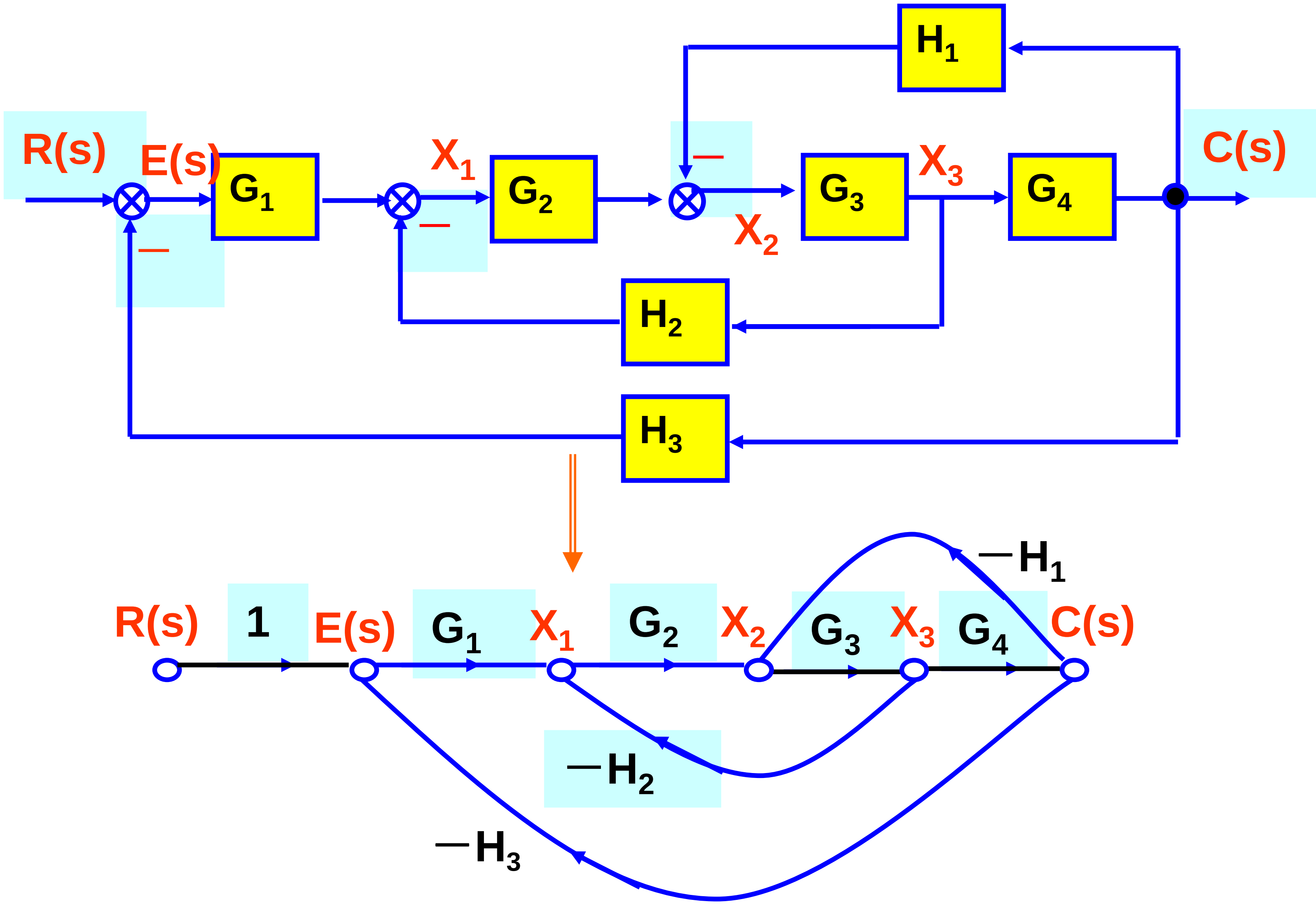
$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

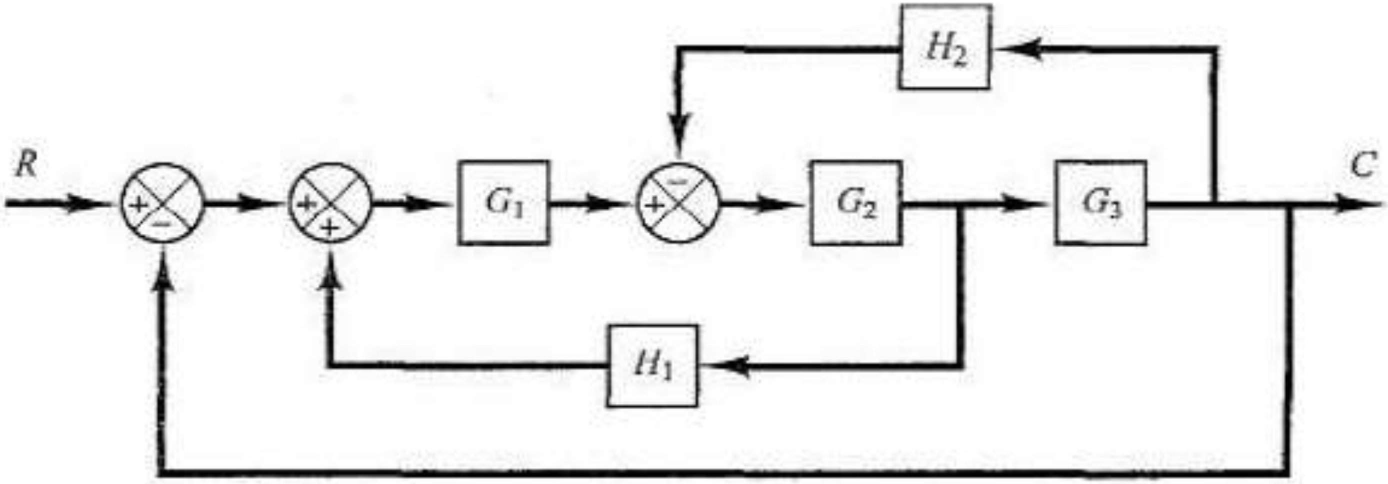
$$C/R = \frac{G_2G_4G_5\left[G_1G_3 + G_6/G_2 + G_6H_1\right]}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

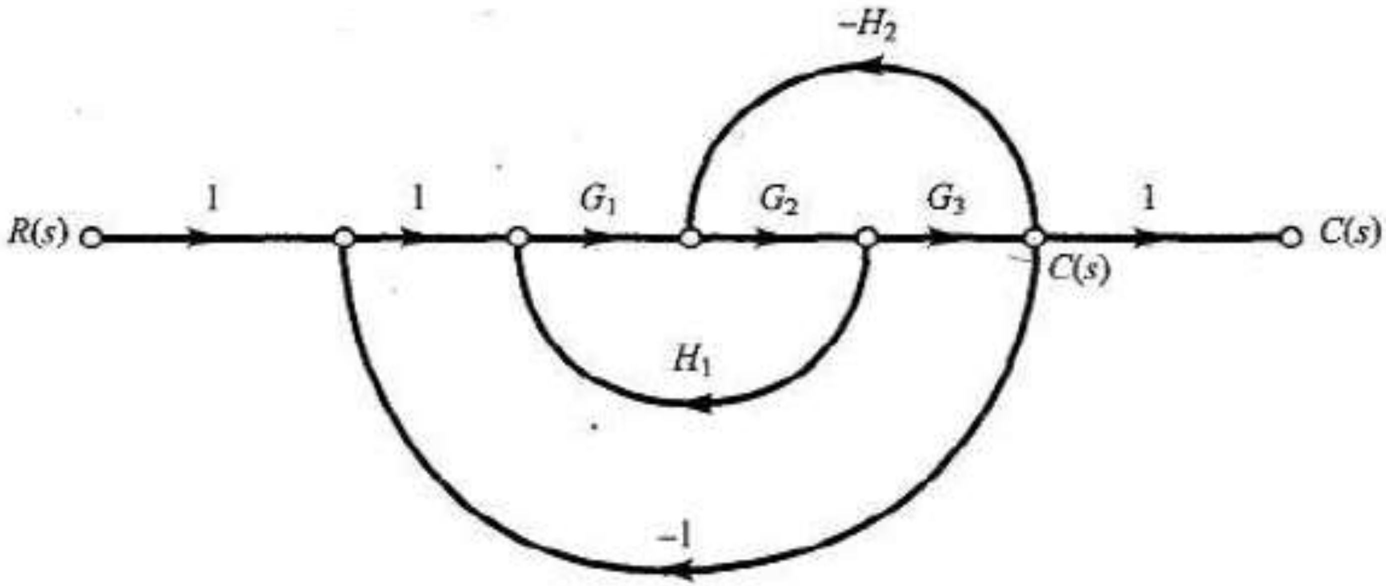
BD to SFG

Block Diagram	Signal Flow Graph
	
	

From Block Diagram to Signal-Flow Graph Models
Example#5




R
C
G1
G2
G3
H1
H2
+
−


R(s)
C(s)
1
1
G1
G2
G3
1
−H2
H1
−1
C(s)

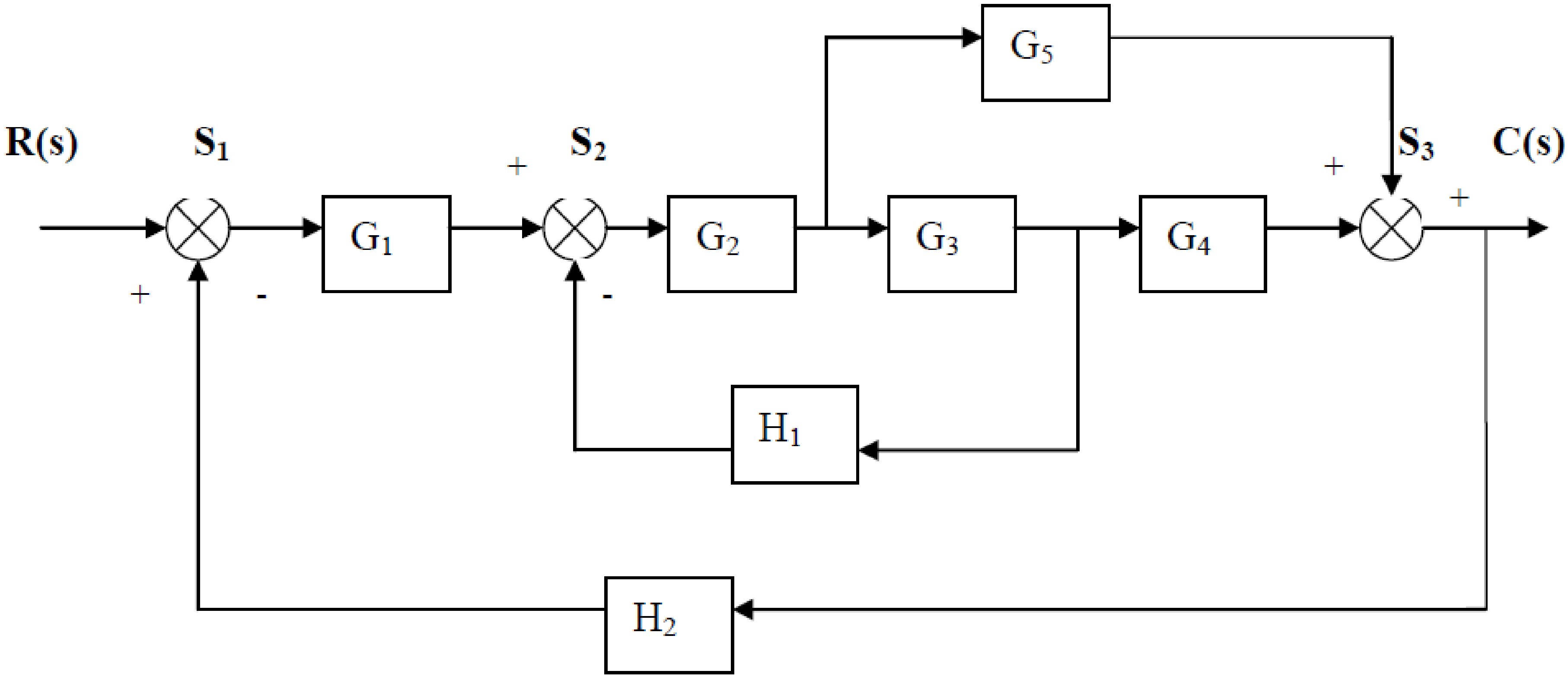
15UEC904
LINEAR CONTROL ENGINEERING

WEEK 1 – EXERCISE PROBLEMS

28.08.2020

Skill Assessment Exercise 1

Find the transfer function of the following system using block diagram reduction techniques



Step :1

Shifting the take off point between the blocks G_2 and G_3 to after the block G_3

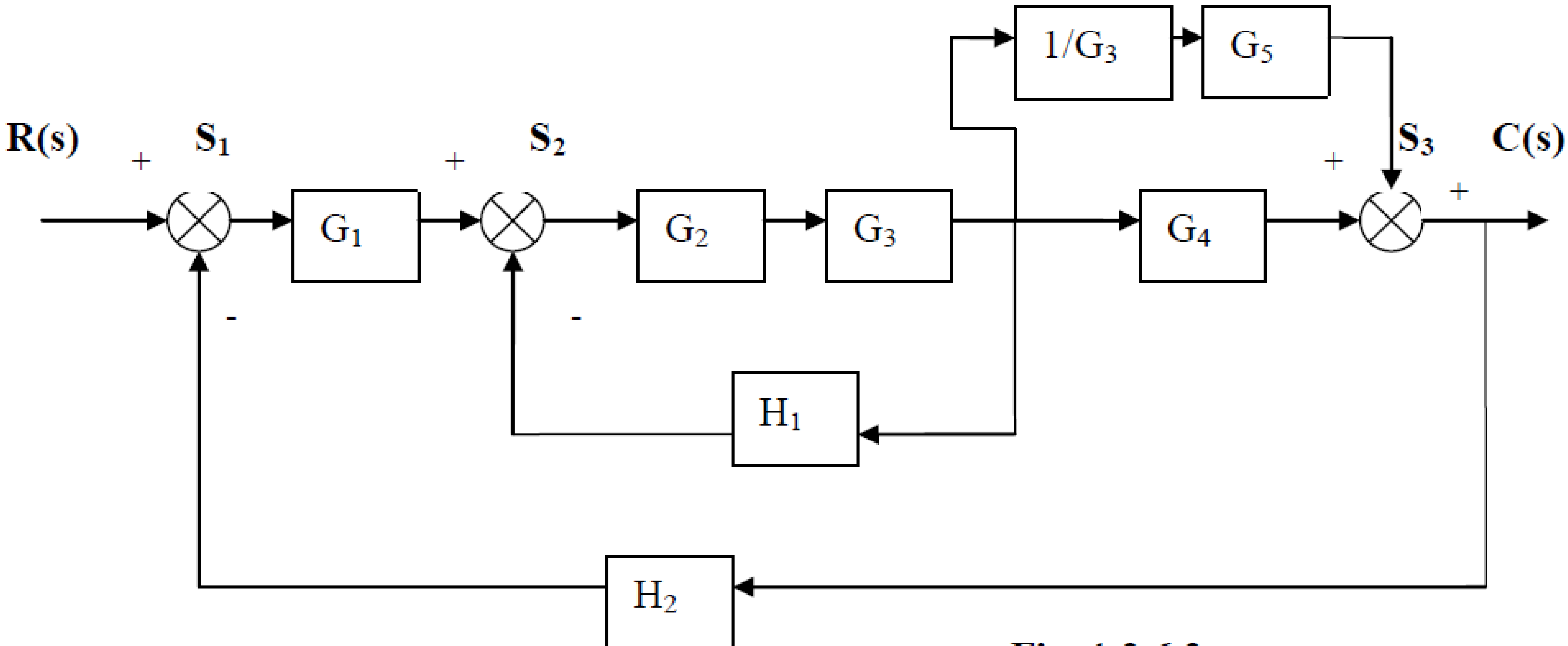


Fig. 1.2.6.2

Step :2

Combine the blocks G_2 and G_3, $1/G_3$ and G_5 which are in series

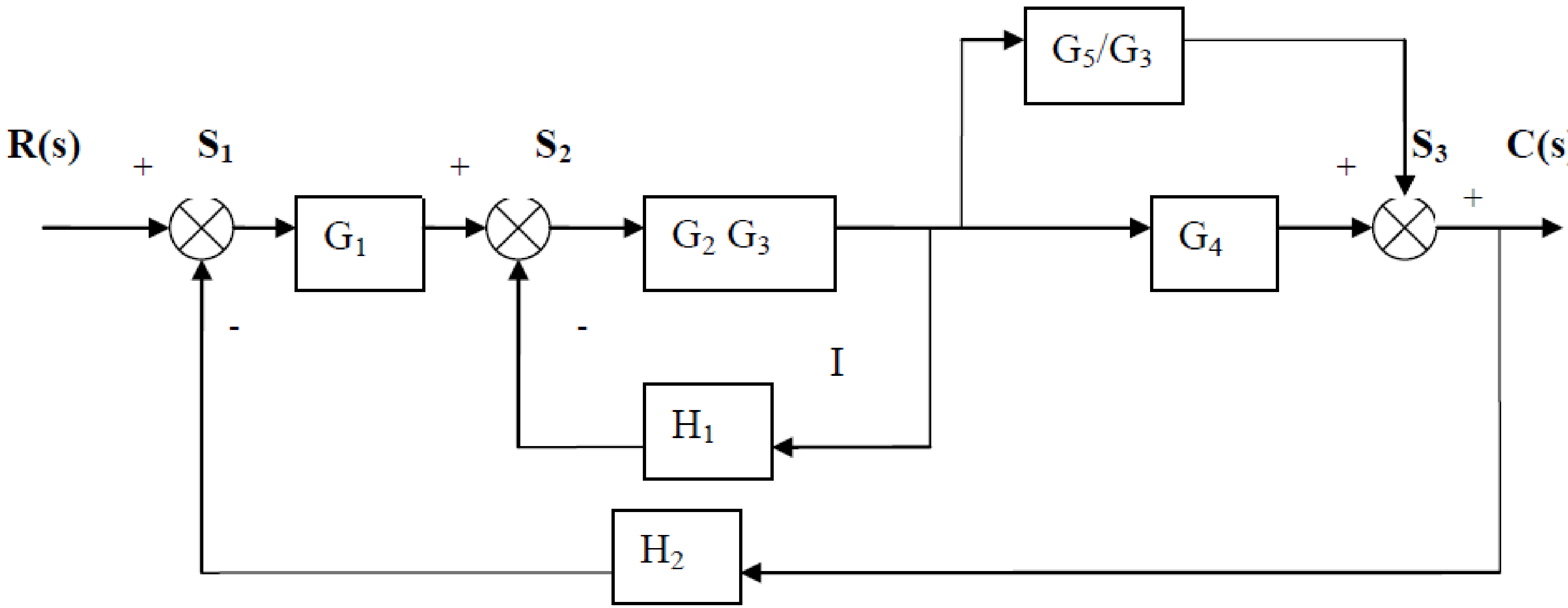


Fig. 1.2.6.3

Step :3

Eliminate feed back loop I, and then combine blocks G_4 and G_5 / G_3 which are in parallel

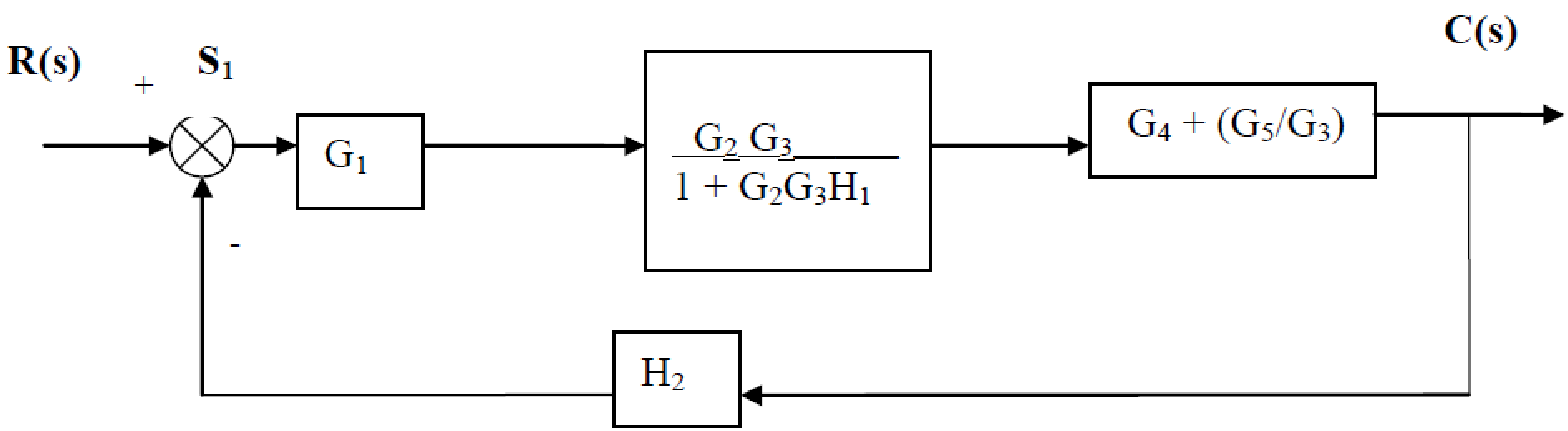


Fig. 1.2.6.4

Step :4

Combine the blocks which are in series

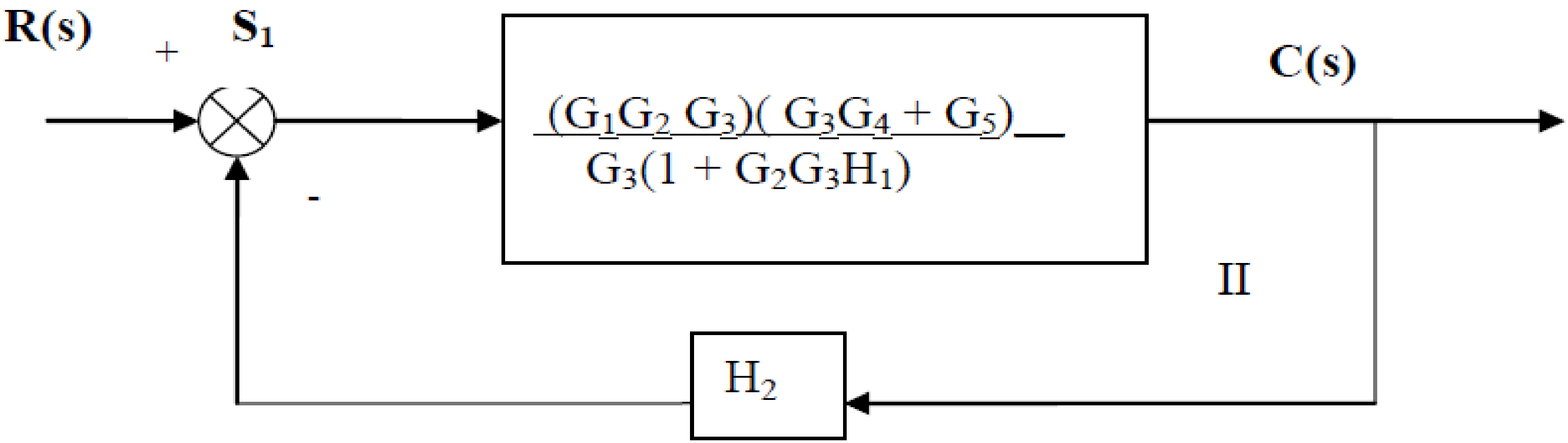


Fig. 1.2.6.5

Step :5

Eliminate the feed back loop II

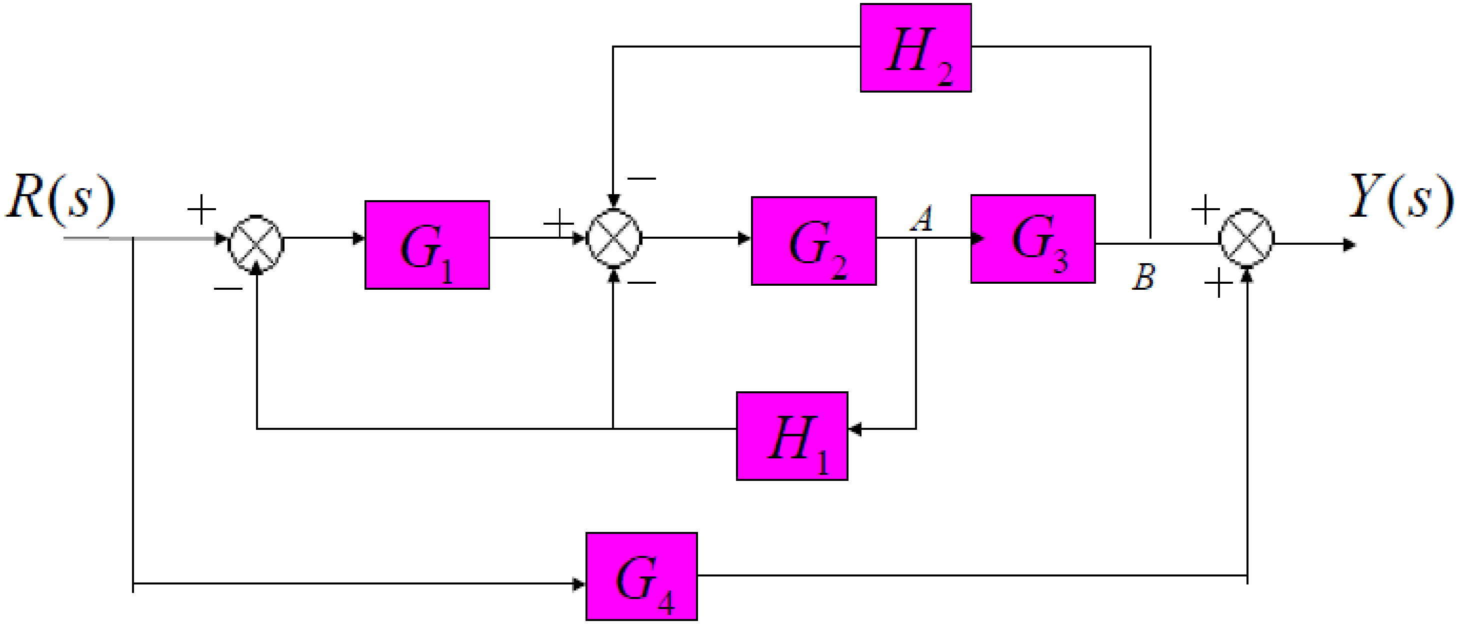
$$\frac{C(s)}{R(s)} = \frac{\dfrac{(G_1G_2G_3)(G_3G_4 + G_5)}{G_3(1 + G_2G_3H_1)}}{1 + \left(\dfrac{G_1G_2G_3(G_3G_4 + G_5)}{G_3(1 + G_2G_3H_1)}\right) H_2}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3\,(G_3G_4 + G_5)}{G_3(1 + G_2G_3H_1) + [G_1G_2G_3\,(G_3G_4 + G_5)]\,H_2}$$

Answer.

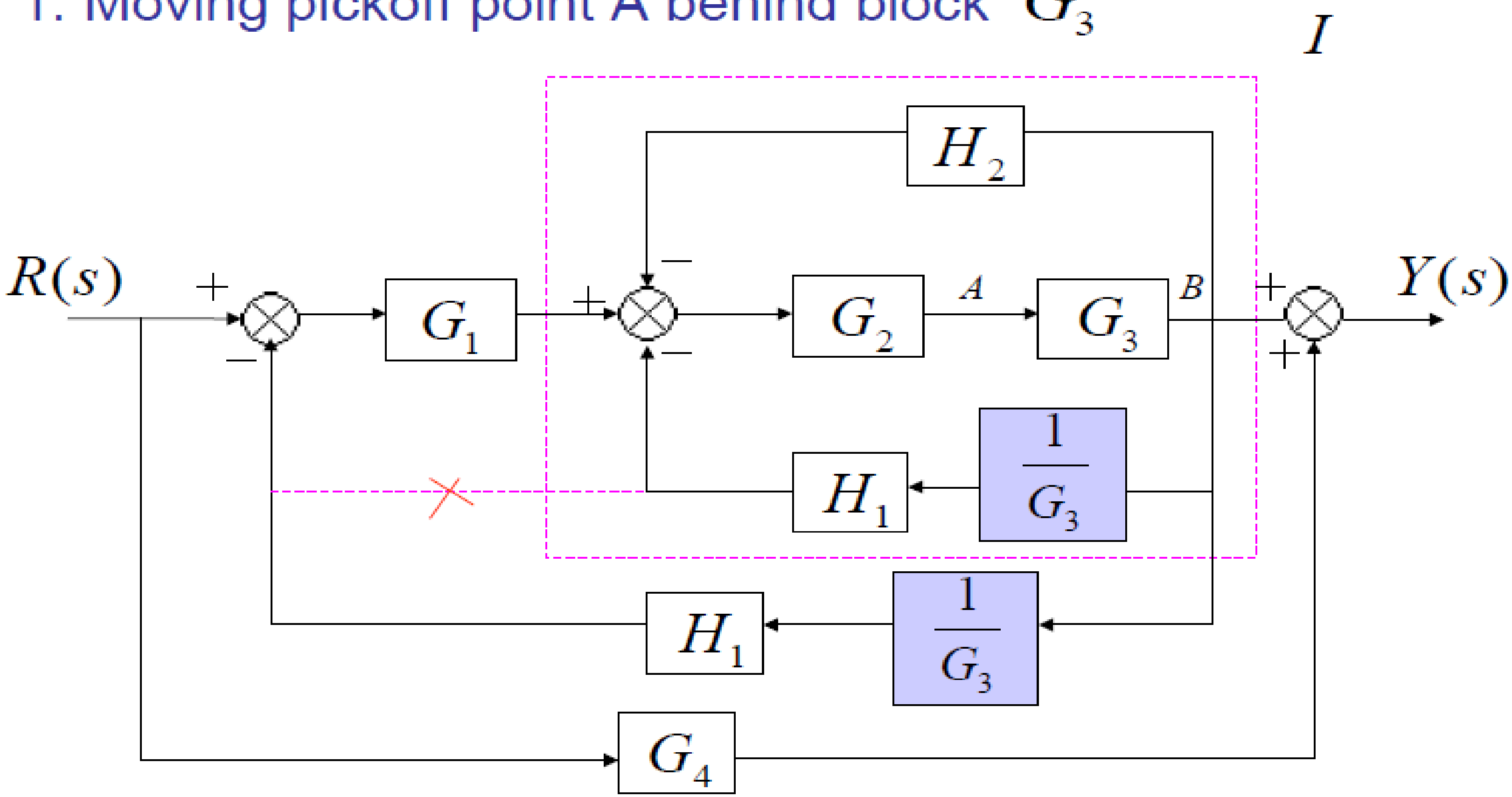
Skill Assessment Exercise 2

Find the transfer function of the following system using block diagram reduction techniques

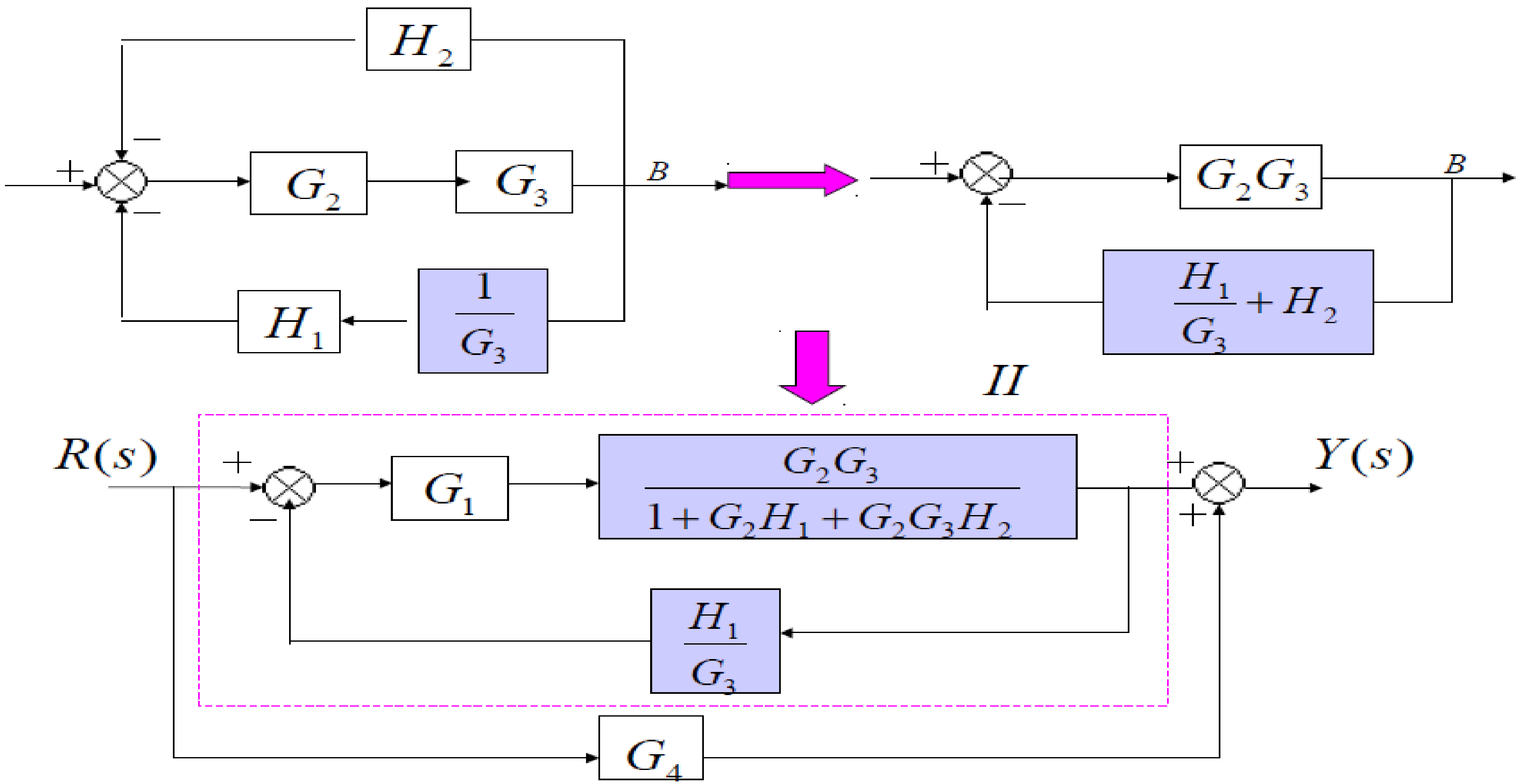


Solution:

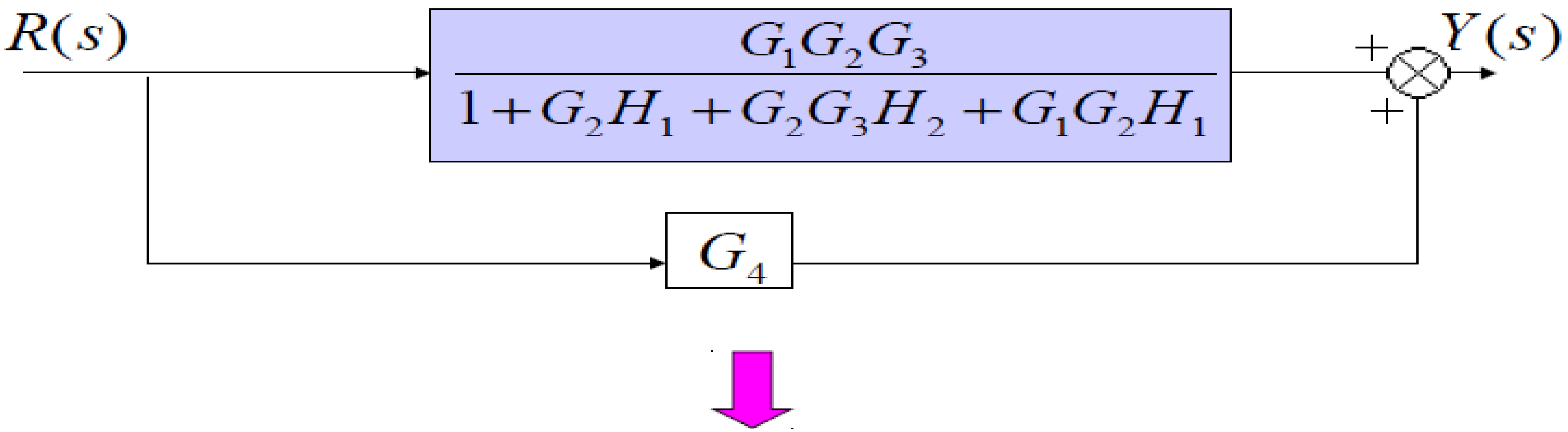
1. Moving pickoff point A behind block G_3



2. Eliminate loop I & Simplify



3. Eliminate loop II



$$T(s)=\frac{Y(s)}{R(s)}=G_4+\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2+G_1G_2H_1}$$

Skill Assessment Exercise 3

Using Mason's gain formula, find the transfer function of the following system represented by signal flow graph

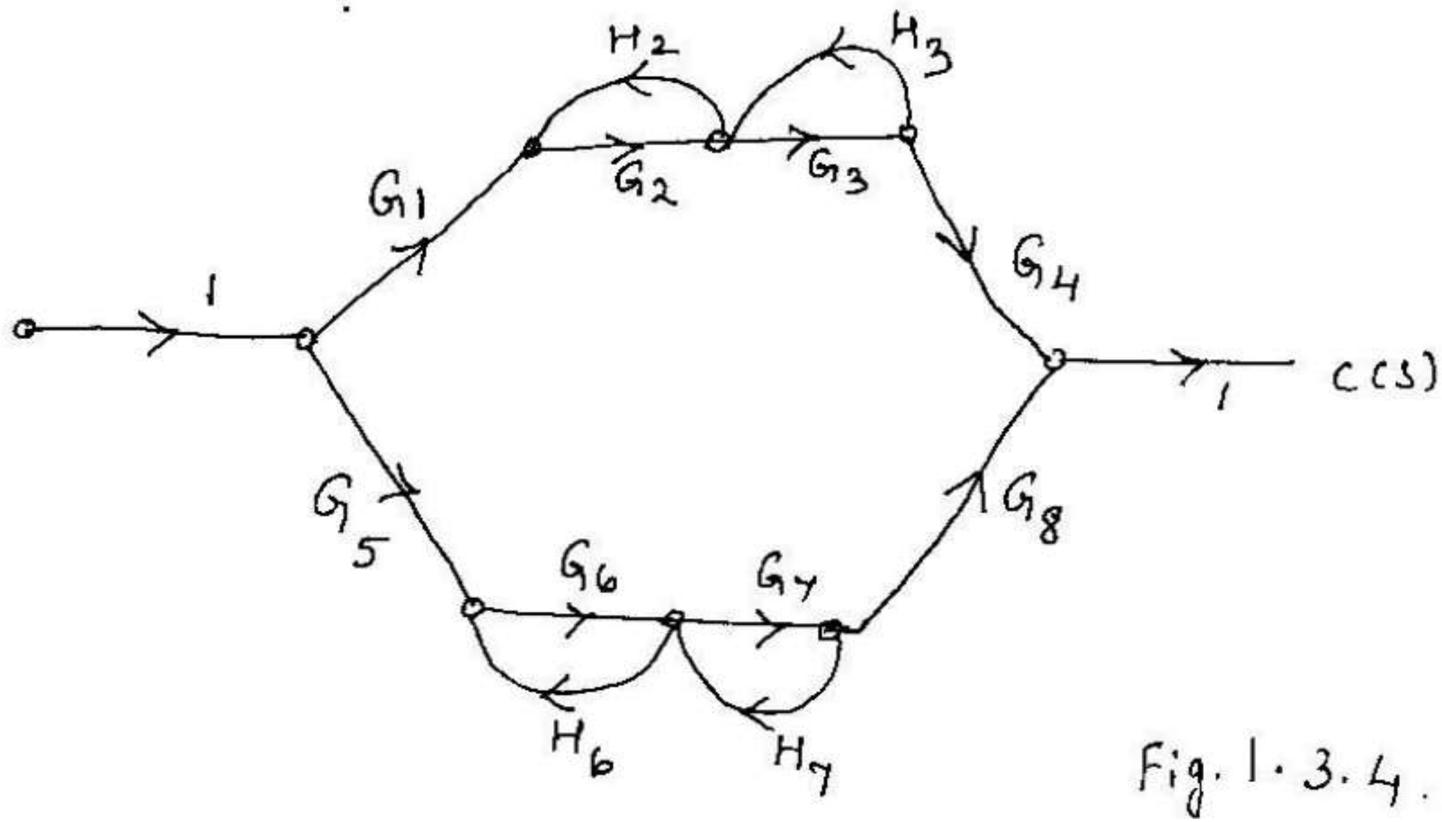


Fig. 1.3.4.

$$\frac{C(S)}{R(S)} = \frac{\sum_{1}^{K} P_K \Delta_K}{\Delta}$$

Here K =2

Forward paths

$P_1 = G_1G_2G_3G_4$

$P_2 = G_5G_6G_7G_8$

Individual loops

$L_1 = G_2H_2$

$L_2 = G_3H_3$

$L_3 = G_6H_6$

$L_4 = G_7H_7$

Two pairs of two non touching loops are there. They are

$L_1 L_3 = G_2H_2 G_6H_6$

$L_2 L_4 = G_3H_3 G_7H_7$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3 + L_2 L_4)$$

$$= 1 - (G_2H_2 + G_3H_3 + G_6H_6 + G_7H_7) + (G_2H_2\, G_6H_6 + G_3H_3\, G_7H_7)$$

$$\Delta_1 = 1 - (G_6H_6 + G_7H_7) = 1 - G_6H_6 - G_7H_7$$

$$\Delta_2 = 1 - (G_2H_2 + G_3H_3) = 1 - G_2H_2 - G_3H_3$$

$$\frac{C(S)}{R(S)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{(G_1G_2G_3G_4)(1 - G_6H_6 - G_7H_7) + (G_5G_6G_7G_8)(1 - G_2H_2 - G_3H_3)}{1 - (G_2H_2 + G_3H_3 + G_6H_6 + G_7H_7) + (G_2H_2\, G_6H_6 + G_3H_3\, G_7H_7)}$$

Ans.

Skill Assessment Exercise 4

Using Mason's gain formula, find the transfer function of the following system represented by signal flow graph

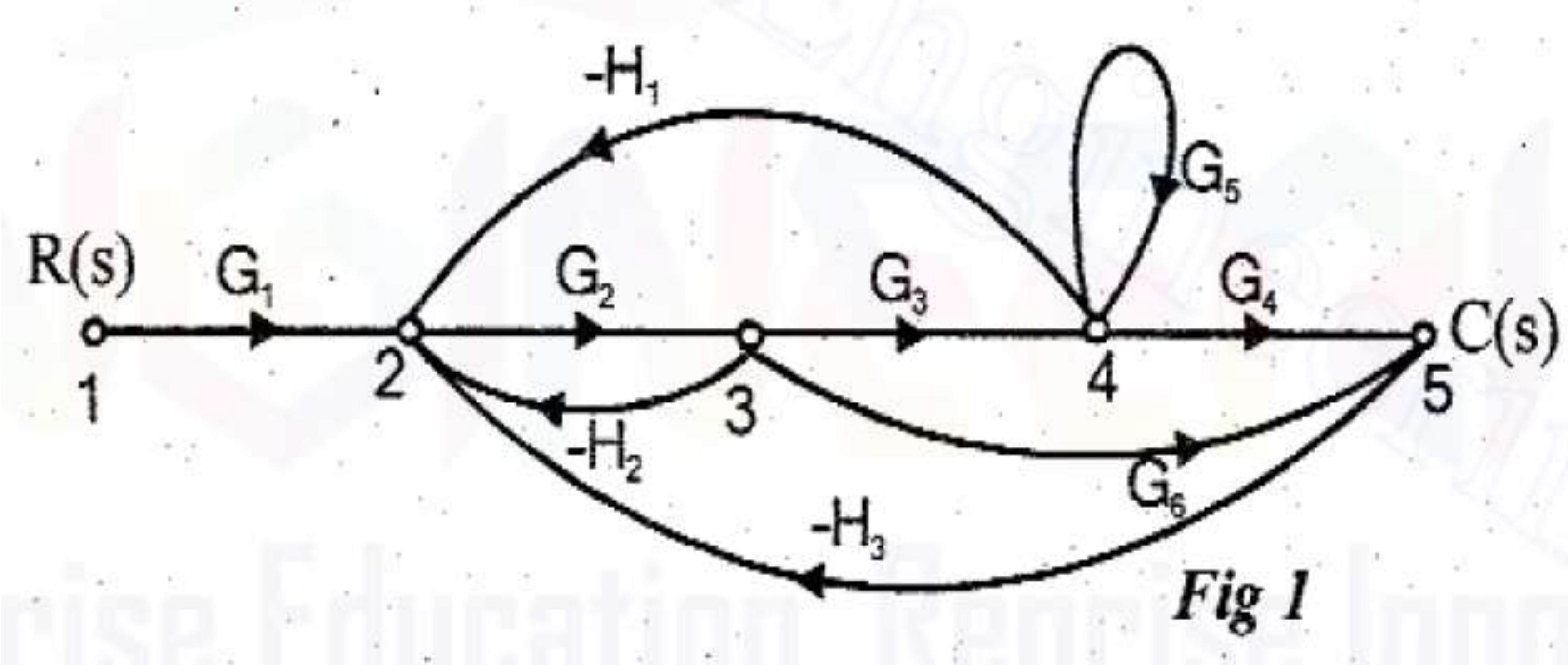


Fig 1

SOLUTION

I. Forward Path Gains

There are two forward paths. $\therefore K = 2$. Let the forward path gains be P_1 and P_2.

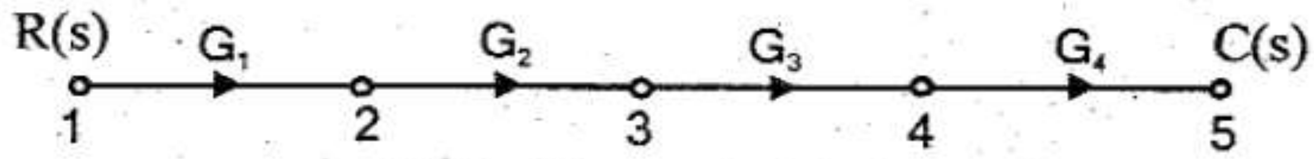


Fig 2 : Forward path-1

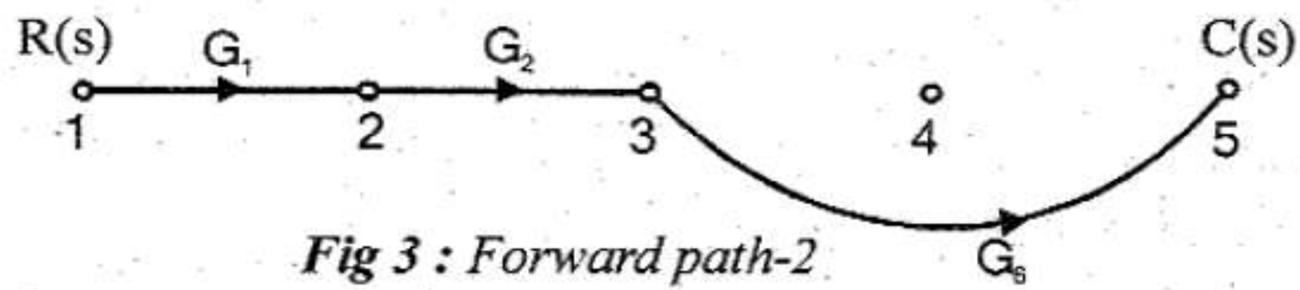


Fig 3 : Forward path-2

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

Gain of forward path-2, $P_2 = G_1G_2G_6$

ndividual Loop Gain

There are five individual loops. Let the individual loop gains be p_{11}, p_{21}, p_{31}, p_{41} and p_{51}.

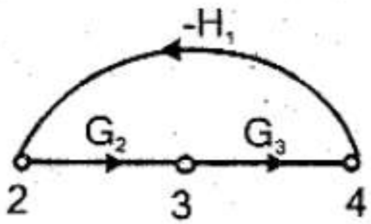


Fig 4 : loop-1

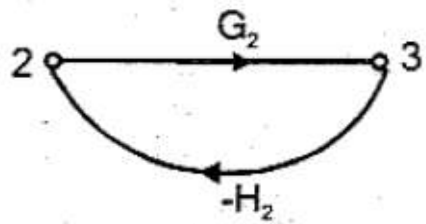


Fig 5 : loop-2

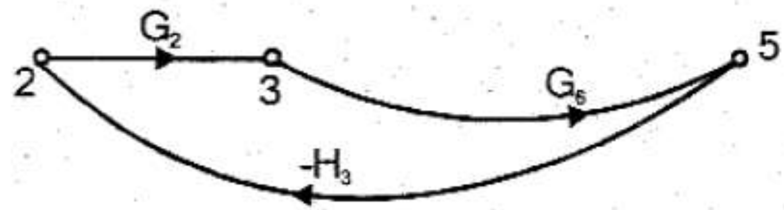


Fig 6 : loop-3

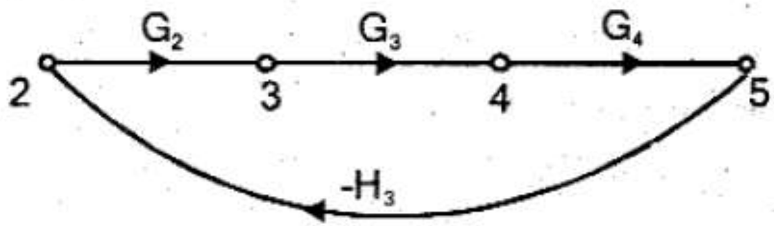


Fig 7 : loop-4

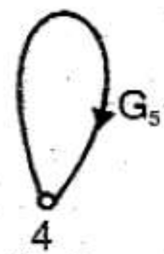


Fig 8 : loop-5

Loop gain of individual loop-1, $P_{11} = -G_2G_3H_1$

Loop gain of individual loop-2, $P_{21} = -H_2G_2$

Loop gain of individual loop-3, $P_{31} = -G_2G_6H_3$

Loop gain of individual loop-4, $P_{41} = -G_2G_3G_4H_3$

Loop gain of individual loop-5, $P_{51} = G_5$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops.

Let the gain products of two non-touching loops be P_{12} and P_{22}.

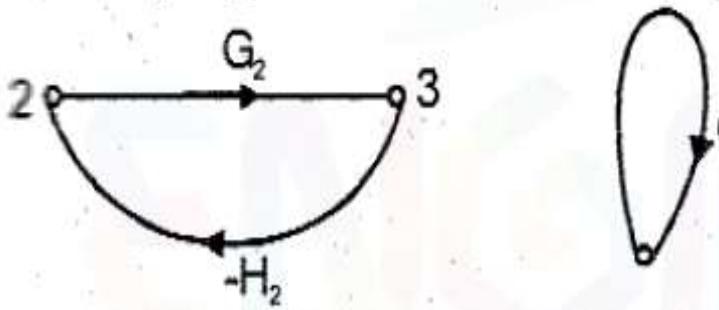


Fig 9 : First combination of two non-touching loops

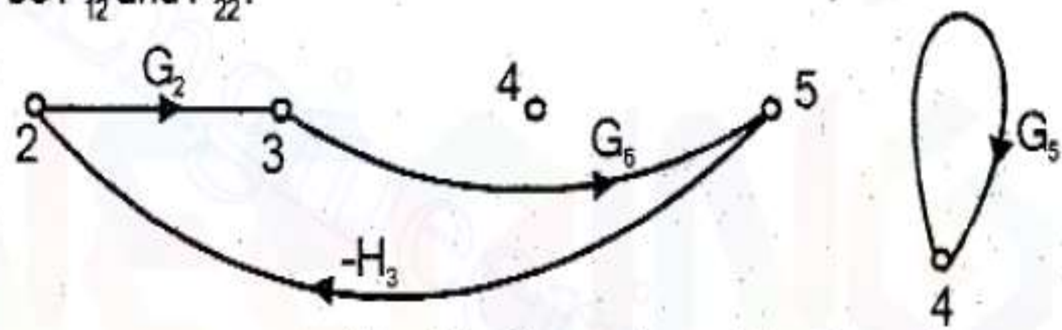


Fig 10 : Second combination of two non-touching loops

Gain product of first combination of two non touching loops $\Big\}$ $P_{12} = P_{21}P_{51} = (-G_2H_2)(G_5) = G_2G_5H_2$

Gain product of second combination of two non touching loops $\Big\}$ $P_{22} = P_{31}P_{51} = (-G_2G_6H_3)(G_5) = -G_2G_5G_6H_3$

Calculation of Δ and Δ_k

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})+(P_{12}+P_{22})$$
$$= 1-(-G_2G_3H_1 - H_2G_2 - G_2G_3G_4H_3 + G_5 - G_2G_6H_3)$$
$$+(-G_2H_2G_5 - G_2G_5G_6H_3)$$

Since there is no part of graph which is not touching forward path-1, $\Delta_1 = 1$.

The part of graph which is not touching forward path-2 is shown in fig 11.

Fig 11

$\therefore \Delta_2 = 1 - G_5$

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$ (Number of forward path is 2 and so K = 2)

$$= \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right] = \frac{1}{\Delta}\left[G_1G_2G_3G_4 \times 1 + G_1G_2G_6(1-G_5)\right]$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + G_2G_3H_1 + H_2G_2 + G_2G_3G_4H_3 - G_5 + G_2G_6H_3 - G_2H_2G_5 - G_2G_5G_6H_3}$$

HAPPY LEARNING